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IV.

39. MIXTURE OF A LIQUID AND ITS VAPOR.—A vapor confined in the presence of the liquid from which it evaporates, soon arrives at its maximum tension and it is then said to be a *saturated* vapor; but a vapor removed from its liquid is, in general, superheated.

The thermodynamic behavior of superheated vapors has been included, at least to a close degree of approximation, in the equations and discussions of the behavior of perfect gases; for superheated vapors are to be regarded as gases which are but slightly more imperfect than the so-called permanent gases.

The physical properties of liquids and their vapors are well understood in a general way from the familiar example of water and steam which may be taken as the type of the mixture to be treated.

The equations for mixtures which we shall arrive at, apply however, not only to mixtures of a liquid and its vapor during evaporation, but they also apply equally to a mixture of any solid and is liquid during liquefaction, such as melting ice and water, or to any chemical decomposition in progress or to the phenomena of dissociation.

The substances heretofore considered have not been regarded as undergoing a change of physical state which is of such a discontinuous nature as vaporization, liquefaction or decomposition, but it has been assumed that the substance under consideration was such that the volume (or any other physical property) of a unit of the given substance was a definite, though perhaps unknown function of its temperature and pressure. But with a mixture of a liquid and its vapor it is otherwise, for in that case the pressure depends directly upon the temperature, so that

$$p=f(t) \quad . \quad . \quad . \quad . \quad . \quad (212)$$

in which f denotes a form of function which can be determined by experiment.

The volume or any other physical property of the mixture evidently cannot in this case be expressed as a function of p and t, for by the help of (212) it would then be possible to eliminate one of these variables and express the volume, etc., in terms of either single variable p or t. The physical state of any substance can not be thus determinately expressed as a function of one independent variable, but requires, as has been previously stated, two independent variables in order to determine its state. Hence any of the previous general equations which can be reduced by help of (212) in such a manner that it will contain but one in-

dependent variable will evidently be useless as applied to a mixture as a whole, though still true as applied to each of the separate homogeneous substances which compose the mixture. We may notice among others that (31), (51), (60), (88) and (94) are useless as applied to mixtures.

40. LATENT HEAT OF EVAPORATION.—

Let l=latent heat of evaporation per unit of weight of liquid at temperature t.

Let x=weight of the vapor contained in a unit of weight of the mixture.

$\therefore$ $1-x$=weight of the liquid in a unit of the mixture.

$$\therefore \quad l=\left(\frac{dh}{dx}\right)_{pt}=t\left(\frac{de}{dx}\right)_{pt} \quad . \quad . \quad . \quad (213)$$

in which the subscripts, as before, denote the constants.

Let v=volume of a unit of weight of the mixture at temperature t.

Let v'=volume of a unit of liquid at t.

Let v''=volume of a unit of vapor at t.

$$\therefore \quad v=(1-x)v'+xv'' \quad . \quad . \quad (214)$$

in which x is independent of t.

$$\therefore \frac{dv}{dx}=v''-v' \quad . \quad . \quad . \quad . \quad (215)$$

in which there is no restriction as to temperature or pressure, hence (215) is also true in case t and p are constant.

$$\therefore \text{ by (213)}, \quad \left(\frac{de}{dv}\right)_{pt}\left(\frac{dv}{dx}\right)_{pt}=\frac{l}{t} \quad (216)$$

$$\therefore \text{ by (215)}, \quad (v''-v')\left(\frac{de}{dv}\right)_{pt}=\frac{l}{t} \quad (217)$$

$$\text{By (42)}, \quad \left(\frac{de}{dv}\right)_{t}=\left(\frac{dp}{dt}\right)_{v}=\frac{dp}{dt} \quad . \quad (218)$$

in case of a mixture. We are at liberty to remove the restriction of constant volume from this last differential co-efficient, because from (211) we shall obtain the same value for it when it is unrestricted as when it is restricted.

$$\therefore \text{ by (217)}, \quad (v''-v')\frac{dp}{dt}=\frac{l}{t} \quad . \quad . \quad (219)$$

in which l is the latent heat of evaporation and $v''-v'$ is the increase of volume of a unit of liquid during evaporation.

Equation (219) is of great importance in the discussion of mixtures, and in it the value of $\frac{dp}{dt}$ is to be derived from (212) the form of which has been determined with great accuracy by the experiments of Regnault.

As a result of these experiments it appears that the latent heat of steam may be expressed with sufficient accuracy for practical purposes by the formula

$$l=\text{a}-\text{b}t \quad . \quad . \quad . \quad . \quad (220)$$

which may be used where convenience demands it, instead of the exact formula (219). In case l is expressed in foot lbs.

$$\text{a}=1109550, \quad \text{b}=540.4.$$

We shall now obtain another equation involving the latent heat l, and the specific heats of the liquid and its vapor.

Let c'=the specific heat of the liquid.

Let c''=the specific heat of the vapor.

Then, by (60), the increments of heat imparted to a unit of the liquid and a unit of its vapor are respectively

$$\frac{dh'}{dt}=\left(k'_p+l'_p\frac{dp}{dt}\right)=c' \ldots \quad (221)$$

$$\frac{dh''}{dt}=\left(k''_p+l''_p\frac{dp}{dt}\right)=c'' \ldots \quad (222)$$

in which the primes refer to the liquid and the seconds to its vapor.

During the evaporation there may be, in the general case, also a change of temperature, hence, the total increment of heat imparted to a unit of the mixture is the sum of the increments imparted to the liquid and vapor respectively, added to the latent heat

$$\therefore dh=c'(1-x)dt+c''xdt+ldx \ldots \ldots (223)$$

In art. 14 it is shown that t^{-1} is an integrating of every value of dh such as (223)

$$\therefore \frac{dh}{t}=de=[c'+(c''-c')x]\frac{dt}{t}+\frac{ldx}{t} \ldots (224)$$

is an exact differential, and fulfills the equation of condition of integrability, which may be expressed as follows:—

$$\frac{c''-c'}{t}=\frac{d}{dt}\left(\frac{l}{t}\right), \text{ or, } c''-c'=\frac{dl}{dt}-\frac{l}{t} \ldots (225)$$

which is independent of x and is the equation involving the latent and specific heats.

It is known that $c' = 772$ foot lbs. nearly, hence by obtaining the terms in the last member of (225) from the approximate equation (220), we find the following approximate value of the specific heat of saturated steam, per degree F.

$$c'' = c' - \frac{a}{t} = \frac{772t - 1109550}{t} \quad \ldots \quad (226)$$

which is a negative quantity at all pressures under a point somewhat above ten atmospheres.

This remarkable result, which has important practical bearings, may be stated thus: When saturated steam is compressed adiabatically, it becomes superheated, the external work of compression being more than sufficient to raise its temperature to the point of saturation due to this higher pressure; but when saturated steam expands adiabatically, some of it condenses, the external work it performs being more than equivalent to the heat emitted from the steam alone during its decrease in temperature.

The specific heat of certain vapors, as ether, are positive, and it appears probable that there is a temperature for each vapor above which its specific heat is positive but below it is negative.

41. Entropy Imparted to a Mixture, etc.—Eliminate $c'' - c'$ between (224) and (225)

$$\therefore de = \left\{ \frac{d}{dt}\left(\frac{l}{t}\right) + \frac{c'}{t} \right\} dt + \frac{l}{t} dx \quad . \quad . \quad (227)$$

$$\therefore de = d\left(\frac{lx}{t}\right) + c' \frac{dt}{t} \quad . \quad . \quad . \quad . \quad . \quad (228)$$

$$\therefore e_2 - e_1 = \frac{l_2 x_2}{t_2} - \frac{l_1 x_1}{t_1} + \int_{t_1}^{t_2} c' \frac{dt}{t} \quad . \quad (229)$$

which is the general expression for the entropy imparted to a unit of the mixture in order to carry it from the state 1 defined by the variables t_1 x_1 to the state 2 defined by the variables t_2 x_2. The last term of (229) cannot be integrated until it is known how c', the specific heat of the liquid, is related to these variables. In the case of water c' is so nearly constant that the result obtained by so regarding it will not lead to sensible error in practice.

$$\therefore e_2 - e_1 = \frac{l_2 x_2}{t_2} - \frac{l_1 x_1}{t_1} + c' \log_e \frac{t_2}{t_1} \quad . \quad . \quad (230)$$

is sensibly exact for a mixture of water and steam.

The last term in (230) being the integral of the last term of (229) expresses the entropy which would be imparted to the unit of water in raising it from t_1 to t_2, or which would be given out by the water in falling from t_2 to t_1. Hence we may consider that this term is the entropy of the sensible heat imparted which causes the change in temperature, and which by the third law is unaffected by the physical condition of the water.

We are enabled by this consideration to find the entropy which must be imparted to a mixture of steam and water in contact with a regenerator or in contact with any metallic conductor which may be regarded as having, by reason of this contact, the same temperature at all times as the mixture. Let the weight of metal in contact with each pound of mixture be y, and its specific heat be c_1,

$$\therefore e_2 - e_1 = \frac{l_2 x_2}{t_2} - \frac{l_1 x_1}{t_1} + (c' + c_1 y) \log_e \frac{t_2}{t_1} \quad (231)$$

is the entropy imparted to the mixture in contact with the metal.

Any equation for steam and water can be corrected so as to apply to the same in contact with metal by substituting $c' + c_1 y$ instead of c'.

We may apply (230) to various special cases of expansion as follows:

1°. During isothermal expansion $t_2 = t_1 = t$ and by (219) or (220) we see that $l_2 = l_1 = l$

$$\therefore \text{by (230)}, h_2 - h_1 = t(e_2 - e_1) = l(x_2 - x_1) \quad . \quad (232)$$

is the total heat imparted during the isothermal expansion of a unit of mixture of steam and water between the states 1 and 2.

2°. During adiabatic expansion $e_2 = e_1 = e$, and if the state 2 is any state on the adiabatic we may conveniently omit the subscripts 2,

$$\therefore \text{by (230)}, \frac{lx}{t} - \frac{l_1 x_1}{t_1} + c' \log_e \frac{t}{t_1} = 0 \quad . \quad (233)$$

is the equation of an adiabatic curve for a unit of mixture of water and steam, in terms of the variables tx, in which $t_1 x_1$ is one point of the adiabatic.

3°. During expansion of the mixture without evaporation or condensation $x_2 = x_1 = x$,

$\therefore$ by (230), $e_2-e_1=x\left(\frac{l_2}{t_2}-\frac{l_1}{t_1}\right)+c'\log\frac{t_2}{t_1}$ . . . . (234)

is the entropy imparted along this curve of no evaporation from t_1 to t_2.

When $x=1$, (234) refers to a unit of steam which is saturated and dry, and the curve expressing its successive states may be called the saturation curve.

42. External Work of a Mixture during Expansion.—If for convenience we let $v''-v'=u$ . . . (235)

$\therefore$ by (219), $u\frac{dp}{dt}=\frac{l}{t}$ . . . . (236)

$\therefore$ by (214) $\frac{dv}{dt}=\frac{dv'}{dt}+\frac{d(ux)}{dt}=\frac{d(ux)}{dt}$. (237)

for v' may be taken as sensibly constant in case of water.

$\therefore$ by (10), $\frac{dw}{dt}=p\frac{dv}{dt}=p\frac{d(ux)}{dt}$ . . . (238)

$\therefore$ $\frac{dw}{dt}=\frac{d(pux)}{dt}-ux\frac{dp}{dt}$ . . . (239)

$\therefore$ by (236), $\frac{dw}{dt}=\frac{d(pux)}{dt}-\frac{lx}{t}$ . . . (240)

Either (238), (239) or (240) may be integrated to find the external work.

1°. In case the expansion is isothermal, p and u are constant

$\therefore$ by (238), $w_2-w_1=pu(x_2-x_1)$ . . . (241)

2°. In case the expansion is adiabatic, we have by (228)

$c'+t\frac{d}{dt}\left(\frac{lx}{t}\right)=0$ $\therefore$ $c'+\frac{d(lx)}{dt}=\frac{lx}{t}$ . . . (242)

$\therefore$ by (240), $dw=d(pux)-d(lx)-c'dt$. (243)

$\therefore$ $w_2-w_1=p_2u_2x_2-p_1u_1x_1-l_2x_2+l_1x_1-c'(t_2-t_1)$ . . . (244)

in which each of the terms in the last member are known in terms of t_2 and t_1 by help of (212), (220), (233) and (236).

3°. Let x be constant during the expansion,

$\therefore$ by (240), $dw=x\left(d[pu]-\frac{l}{t}dt\right)$. (245)

$\therefore$ by (220), $dw=x\left(d[pu]-\frac{a}{t}dt+bdt\right)$ . . . (246)

$\therefore w_2-w_1=x\left(p_2u_2-p_1u_1-a\log_e\frac{t_2}{t_1}+b[t_2-t_1]\right)$ . . . (247)

from which we obtain the work along the saturation curve by making $x=1$.

43. Internal Energy and Total Heat Imparted to a Mixture.—

By (228), $dh=tde=c'dt+td\left(\frac{lx}{t}\right)$. (248)

$\therefore$ $dh=c'dt+d(lx)-\frac{lx}{t}dt$ . . (249)

But by (236), $\frac{lx}{t}dt=uxdp=d(pux)-d(ux)$ . . . (250)

$\therefore$ by (237), $\frac{lx}{t}dt=d(pux)-pdv$. (251)

$\therefore$ by (249), $dh=c'dt+d(lx)-d(pux)+pdv$ . . . (252)

Either (248), (249) or (252) may be integrated to obtain the total heat imparted, which can also be found from (223).

Equation (4) may be written by (10) in the form $dh=di+pdv$, and by comparing this with (252), we have

$di=c'dt+d(lx)-d(pux)$. (253)

$\therefore$ $i_2-i_1=c'(t_2-t_1)+l_2x_2-l_1x_1-p_2u_2x_2+p_1u_1x_1$ . . . (254)

which is independent of the kind of expansion, as it should be by art. 15.

The total heat imparted is to be obtained in any case by adding (254) to the external work obtained in art. 42.

The total heat imparted during expanpansion along a curve of no evaporation is so important in its practical bearings and so simple in its expression that we shall now obtain it. In this case $x_2=x_1=x$. Substitute the value of l taken from (220) in (248)

$\therefore$ $dh=c'dt-ax\frac{dt}{t}$ . . (255)

$\therefore$ $h_2-h_1=c'(t_2-t_1)-ax\log_e\frac{t_2}{t_1}$ (256)

from which by making $h_2-h_1=0$, we can find the value of x such that no heat need be supplied in order that x may have the same value at t_1 as at t_2.

44. Empirical Formulae for Steam.—It appears from the indicator cards of steam engines and other experiments on the expansion of steam that the curves of expansion can be represented with sufficient exactness for practical purposes by an equation of the following form:

$pv^n=c$ . . . . (257)

which has the form previously employed in case of perfect gases, but the constants in (257) have values for steam which differ from those of perfect gases.

In case of superheated steam it has been shown by experiment that even to the point of saturation.

$$n=\tfrac{4}{3}=1.3333 \quad . \quad . \quad (258)$$

is a close approximation. We wish to direct especial attention to this important experimental result since it has been widely misstated, and hence shall quote authority for it at some length by translating a paragraph found on p. 448 of *Théorie Méchanique de la Chaleur*, par G. Zeuner, traduit de l'Allemand, par Arnthal et Cazin, Paris, 1869. This paragraph is found in an addition made by these translators—"Zeuner supposes, *a priori*, that the equation of the adiabatic curve for superheated steam, coincides with that of gases; only he replaces the constant $n=1.41$, which applies to gases by $n=\frac{4}{3}$. He regards this hypothesis as sufficiently justified by the exactness of the results which he deduces from it. We are constrained to say that it agrees sufficiently well with the recent experiments which M. Hirn has made in conjunction with one of us.* Though our experiments have not shown that n is rigorously constant, yet the numbers which we have deduced for that quantity vary so little from $\frac{4}{3}$ that Zeuner's constant is acceptable in technical applications."

Rankine refers to the same experiments on p. 320 of his *Steam Engine* as authority for $n=1.3$ for steam in a perfectly gaseous state.

McCulloch on p. 234 of his *Treatise on the Mechanical Theory of Heat* makes the statement that $n=\frac{4}{3}$ for saturated steam, referring to the same experiments as authority for the statement, which taken in connection with the rest of the paragraph has been taken to mean that $n=\frac{4}{3}$ for a mixture of steam and water during adiabatic expansion. This is a mistake, as Zeuner shows on p. 332 of the book above referred to, that we are justified in taking as a mean value,

$$n=1.135 \quad . \quad . \quad . \quad . \quad (259)$$

for adiabatic expansion when the steam is saturated and dry at the beginning of the expansion and expands within the limits usual in steam engines.

In case of a mixture of steam and water, Zeuner further shows on p. 335, that for adiabatic expansion

$$n=1.035+\tfrac{1}{10}\,x_1 \quad . \quad . \quad . \quad . \quad (260)$$

in which x_1 denotes, as previously, the fraction of the unit of mixture which at the beginning of the expansion is in the form of steam.

It is seen that (259) is a case of (260). Rankine was the first who proposed the use of a formula of the form of (257), and he proposed in ordinary practice to

let $$n=\tfrac{10}{9}=1.1111 \quad . \quad . \quad . \quad . \quad (261)$$

by comparing which with (260) it will be found that it corresponds to the case in which $x_1=\frac{3}{4}$ nearly.

Besides adiabatic expansion, the case most necessary to treat is that in which the steam is kept saturated and dry. The importance of this case arises from the fact that in a jacketed steam cylinder while it may be very possible to supply heat so as to prevent condensation during the stroke, it would be hardly possible to super-heat the steam in course of the stroke owing to the fact that steam is a poor conductor of heat. Zeuner shows on p. 301 of the book before quoted that

$$n=1.0646 \quad . \quad . \quad . \quad . \quad (262)$$

in case the steam is kept saturated and dry during expansion.

Rankine proposed in his *Steam Engine* for expansion in jacketed cylinders

$$n=\tfrac{17}{16}=1.0625 \quad . \quad . \quad . \quad . \quad (263)$$

which differs little from (262).

To recapitulate these results approximately,

$n=1.41=\frac{7}{5}$, for adiabatic of perfect gases,
$n=1.333=\frac{4}{3}$, for adiabatic of steam gases,
$n=1.135=\frac{17}{15}$, for adiabatic of wet steam,
$n=1.0645=\frac{17}{16}$, for saturation curve of steam,
$n=1.000$, for isothermal of perfect gases.

The relation between these curves is shown in Fig. 6, in which it is seen that the greater the value of n the steeper the slope of the curve. The curves so nearly coincide when drawn to scale that strict

* *Mémoire sur la détente de la vapeur d'eau surchauffée*, par MM. Hirn et Cazin. (*Annales de Chemie et de Physique*, 4e série, tome X).

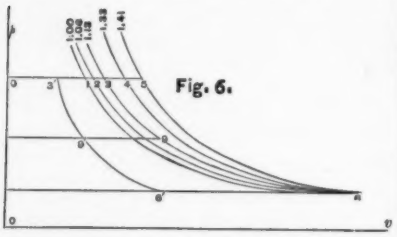


Fig. 6.

accuracy has not been sought in the figure.

Suppose for example that a volume of dry and saturated steam represented by 03 expands adiabatically in a cylinder to 6; then as previously stated in art. 40, part of it is condensed during the expansion. This fact has been observed experimentally by Hirn, and is now universally admitted. The work performed during the admission of steam from the boiler, and during its expansion is represented by the area 0367o. If the cylinder should be employed as a compressor, the water which was condensed during expansion would be re-evaporated during the compression along 63, and the total work required to drive the steam into the boiler is also represented by the area 0367o. But suppose we should, instead of driving back the entire mixture, separate the condensed water at 6 from the steam, and drive back the steam only. Neglecting the volume of the water so separated, the steam will on compression become superheated (as it has no water to re-evaporate), and will be compressed along 64, so that the work to be performed in forcing it back into the boiler will be represented by 0467o, which is an amount larger than was performed during the admission of the steam to the cylinder and during its expansion.

Certain new types of steam engines have been recently proposed by Prof. Thurston* apparently on the supposition that the second law of thermodynamics does not apply to steam. As no known facts are at variance with this law, such a supposition is inadmissible.

In particular, it was contemplated that one of these types of steam engines should have essentially the cycle above described, the proposer quoting certain authorities to show that the curve 64 lies on the other side of 63 from that shown in Fig. 6.

If it is possible to accomplish the result proposed in these engines, then the second law, which is apparently well grounded, is not tenable, and we must not only set aside the results of that most accurate of modern experimentalists, Regnault, as untrustworthy, but we must also regard all who have made experiments upon the specific heat of steam as having arrived at erroneous results.

45. Adiabatics for Steam. — These curves are given by (233), or by (257) and (260), but these equations are somewhat complex for practical purposes. A practical simplification has been effected which, together with other matters of importance, will now be detailed.

Suppose that the three adiabatics be drawn for a unit of mixed steam and water starting from the temperature t_1, in which x_1, x_1', x_1'' are fractions expressing the initial proportions of steam in the three cases. Then by (233),

$$\left.\begin{aligned} \frac{lx}{t}-\frac{l_1x_1}{t_1}&=c'\log_e\frac{t_1}{t}\\ \frac{lx'}{t}-\frac{l_1x_1'}{t_1}&=c'\log_e\frac{t_1}{t}\\ \frac{lx''}{t}-\frac{lx_1''}{t}&=c'\log_e\frac{t_1}{t}. \end{aligned}\right\} \quad (264)$$

$$\therefore \frac{x-x'}{x''-x'}=\frac{x_1-x_1'}{x_1''-x_1'} \quad (265)$$

which is a relation between any three adiabatics. In order to render this of practical use, let $x_1'=0$, in which case the mixture initially contains no stream, and the adiabatic sensibly coincides with the axis of no volume; also let $x_1''=1$, in which case the adiabatic is that of steam initially saturated and dry (259),

$$\therefore \text{ by (265)}, \quad \frac{x-x'}{x''-x'}=x_1 \quad (266)$$

Now by (214), $v=w+ux$

$$\left.\begin{aligned} v'&=w+ux'\\ v''&=w+ux'' \end{aligned}\right\} \quad (267)$$

$$\therefore \text{ by (266)}, \quad \frac{v-v'}{v''-v'}=x_1 \quad (268)$$

in which w is the volume of a unit of

* Journal of the Franklin Institute, Vol. 104, pp. 252, 316, 389.
Proceedings of the Am. Assoc. Advance Science, 1877, p. 85.
Hist. of the Growth of the Steam Engine. New York, 1878, p. 484, *et seq.*

water, v, v' v'' are the volumes of the respective mixtures at any given temperature t, or what by (212) is the same thing at any given pressure p. Hence to construct the adiabatic corresponding to any initial value of x_1: Suppose, as in Fig. 6, that the adiabatic 396 for steam initially saturated and dry has been constructed, as also the adiabatic for water, which may be taken as approximately vertical, then by (268), we have approximately,

$$v = x_1 v'' \quad . \quad . \quad . \quad . \quad (269)$$

In Fig. 6 the adiabatic 3′9′6′ is drawn for the case in which $x_1 = \frac{1}{2}$.

The same relation holds between the various curves of no evaporation, as appears by combining equations (267) together; so that to obtain the curve for which x has a given value, it is only necessary to draw that for which $x' = 1$, as obtained by (262), and then subdivide the abscissas in the manner expressed by (269).

46. Work of Expansion of Steam.—This can be found for the different cases by art. 42, but the convenience of the empirical formulae (257), etc., in art. 44, have led to the frequent use of expressions for the work derived from them.

By (10) and (257), $dw = pdv = cv^{-n}dv$

$$. \quad . \quad . \quad . \quad (270)$$

$$\therefore \; w_2 - w_1 = \frac{c}{1-n}(v_2^{1-n} - v_1^{1-n}) = \frac{p_1 v_1 (1-r)}{n-1} \quad . \quad . \quad . \quad (271)$$

in which by (186) $v_2^{1-n} = r v_1^{1-n}$.

When, however, $n = 1$, the curve is a rectangular hyperbola, and we have from (270)

$$w_2 - w_1 = p_1 v_1 \log \frac{v_2}{v_1} \quad . \quad . \quad . \quad (272)$$

The work performed along the different curves does not differ greatly, and hence (272) is frequently used for computing the external work, especially any leakage through the valves causes this result to be more nearly correct.

The total heat supplied along these different empirical curves is, however, quite different, and it cannot be computed from the curves as they appear on the indicator card.

47. Adiabatic Flow of Mixed Steam and Water.—

By (10) $dw = pdv = d(pv) - vdp \quad . \quad . \quad . \quad (273)$

$$\therefore \text{ by (131)}, \; d\left(\frac{\mathrm{v}^2}{2g}\right) = -vdp = dw - d(pv) \quad . \quad . \quad . \quad . \quad (274)$$

$$\therefore \text{ by (243)}, \; d\left(\frac{\mathrm{v}^2}{2g}\right) = d(pux) - d(lx) - c'dt - d(pv) \quad . \quad . \quad . \quad (275)$$

Integrate between the states 1 and 2, and reduce by the following equations derived from (214)

$$v_1 = v' + u_1 x_1, \; v_2 = v' + u_2 x_2,$$

$$\therefore \frac{\mathrm{v}^2}{2g} = c'(t_1 - t_2) + l_1 x_1 - l_2 x_2 + v'(p_1 - p_2) \quad . \quad . \quad . \quad . \quad (276)$$

which result after neglecting the term $v'(p_1 - p_2)$ can be still further reduced,

$$\text{for by (233)}, \; l_2 x_2 - \frac{t_2}{t_1} l_1 x_1 - c' t_2 \log \frac{t_2}{t_1} \quad . \quad . \quad . \quad . \quad (277)$$

$$\therefore \frac{\mathrm{v}^2}{2g} = \left(c' + \frac{l_1 x_1}{t_1}\right)(t_1 - t_2) + c' t_2 \log_e \frac{t_2}{t_1} \quad . \quad . \quad . \quad . \quad (278)$$

which is the fundamental equation of adiabatic flow for the mixture.

In case $x_1 = 0$ we have the expression for the flow of water from a boiler, in which case $t_2 = 212°$ F, and the term $v'(p_2 - p_1)$ must not be neglected, as it must not be whenever x_1 is small.

In order to determine the state of steam after efflux we must find the total increment of internal energy after the energy of motion has been transformed into heat, which may be done in several ways, but most readily from the consideration that since no heat is received during efflux,

$$di + dw + vdp = 0 \quad . \quad . \quad . \quad (279)$$

$$\therefore \text{ by (273)}, \; di + d(pv) = 0 \quad . \quad . \quad . \quad (280)$$

$$\text{But, } v = v' + ux \quad \therefore \; d(pv) = d(pux) \quad (281)$$

$$\therefore \text{ by (253) and (280)}, \; c'dt + d(lx) = 0 \quad (282)$$

$$\therefore \; c' + x\frac{dl}{dt} = -l\frac{dx}{dt} \quad . \quad . \quad . \quad . \quad (283)$$

$$\therefore \text{ by (220)}, \; 772 + 540x = -l\frac{dx}{dt} \quad . \quad (284)$$

from which it appears by (220), that as the temperature decreases evaporation occurs, *i.e.* that when dt is negative dx must be positive.

Equations (279) to (280) are the equations of the isodynamic curves of steam. Zeuner states that the isodynamic curves

of wet steam are represented by (257) when

$$n=1.0456 \quad . \quad . \quad . \quad . \quad (285)$$

Hirn deduces from his experiments that in case of superheated steam $n=1$. A comparison of these curves with the curves of no evaporation and saturation curves shows that the result deduced from (284) is correct, and that in case the steam is saturated before efflux it is superheated after efflux, although it is not so within the orifice, before its energy of motion has been transformed into heat. This result can be observed, as a conical surface separating the cloudy (wet) steam from the transparent (superheated) steam can be seen at an orifice when steam is discharged at a high initial pressure.

48. Steam Injectors and Ejectors.—A steam injector consists essentially of a condensation chamber into which flows a jet of steam and also a supply of water, and out of which flows a jet of water.

Let the steam jet flow from a boiler in which the state is denoted by subscript 1. Let the state of the water at its entrance into the chamber be denoted by the subscript 0, and let 2 denote the state of the jet of water flowing from the chamber. The velocity of the water flowing into the chamber may be neglected, and its pressure p_0 should be computed by taking the algebraic sum of the atmospheric pressure and the pressure due to the head of the reservoir from which the supply comes, above or below the chamber.

Consider first the jet of wet steam supplied by the boiler, which is condensed in the chamber and flows out in the jet of water. If v is the velocity of jet of water, then the total energy lost, per unit of steam supplied, in its passage through the chamber is evidently

$$i_1-i_2+p_1v_1-p_2v_2-\frac{v^2}{2g} \quad . \quad (286)$$

for the steam carries with it into the chamber its initial internal energy i_1, and the external work p_1v_1, which is expended upon by the boiler in forcing it into the chamber, and in going out of the chamber it carries out internal energy i_2 and does external work p_2v_2, besides its energy of motion due to the velocity v. The loss of energy expressed by (286) is that imparted to the water it meets in the chamber. The energy gained, by each unit of the water supplied, between its entrance into the chamber and its efflux from the chamber is expressed similarly by

$$i_0-i_2+p_0v_0-p_2v_2-\frac{v^2}{2g} \quad . \quad (287)$$

Now suppose that each unit of water flowing out of the chamber is composed of a fraction of a unit y which came from the boiler, and the rest of the unit $1-y$, which came from the reservoir. Hence (286) multiplied by y and added to (287) multiplied by $1-y$ is equal to the total heat per unit of efflux of water imparted to or taken from the chamber by conduction or otherwise than through the water and steam. This under ordinary circumstances may be taken as zero,

$$\therefore \frac{v^2}{2g}=y(i_1-i_0+p_1v_1-p_0v_0) \\ -(i_2-i_0+p_2v_2-p_0v_0) \quad . \quad (288)$$

By (254), $i_1-i_0=c'(t_1-t_0)+l_1x_1-p_1u_1x_1$, (289)

for $x_0=0$; and similarly in i_2-i_0, $x_2=0$,

$$\therefore \frac{v^2}{2g}=y[c'(t_1-t_0)+l_1x_1+(p_1-p_0)v'] \\ -c'(t_2-t_0)+(p_2-p_0)v' \quad . \quad (290)$$

This is the fundamental equation of the injector, and in it all the quantities except v, y and t_2 may be considered to be fixed in any given case by circumstances over which we have no immediate control. Indeed t_2 is also partially independent of control for the injector will not work, *i.e.* complete condensation will not take place, if t_2 is near boiling point at the pressure inside the chamber. But over y it would at first appear that we have complete control, for the proportions of steam and water can be regulated by suitable cocks, but such is not in reality the case, for by sufficiently increasing y, t_2 will be increased beyond the boiling point within the chamber. It then appears that t_2 is dependent on y.

Were it possible to find what function t_2 is of y, we should be able to determine what value of y would, in any given case, give a maximum value to v. But it is evident that it is impossible to find out what this function is in general, because it depends upon the construction, arrangement, relative size and position of the tubes conveying steam and water.

It would seem possible to solve the problem in the case when the greatest velocity is given to the water which can theoretically be communicated to it, but the solution would be of no practical importance in case the injector is employed, as it usually is, to supply the feed water to a steam boiler. For in this case all of the energy which is drawn from the boiler in order to work the injector is restored again to the boiler in the internal energy of heat and motion of the feed water, except that expended in raising the feed water from the level of the reservoir to that of the boiler, as is evident from the fact that no energy has been expended except upon the feed water. Hence it is of no consequence how much steam is used in driving the injector since it all returns to the boiler. It thus appears that a boiler injector is a machine whose efficiency is unity, and hence it is impossible that any other apparatus can work more efficiently. If, however, the injector be used simply as a pump for raising water from one level to another, it is a matter of prime importance to investigate the relation of t_2 to y, for in this case the energy expended in heating the water is lost, and the efficiency depends upon the energy of motion communicated to the water. In any case (290) enables us to compute the energy of motion imparted to the water when we have observed t_2 the temperature of the effluent water.

The action of the steam injector is more prompt and energetic when the temperatures t_0 and t_2 are sufficiently low to insure rapid condensation.

To determine the limiting value of t_2, above which it is impossible for the injector to work, we must find the temperature t' of saturated steam at the pressure p_0, which is the hydrostatic pressure within the condensation chamber, $\therefore t_2 < t'$. In practice it is found that t_2 must be from 12° to 20° F. below t' in order to insure a sufficiently rapid condensation.

The efficiency of the pumps which have thus far been constructed on the principle of the injector has been small, but there is another way in which this apparatus is employed, viz: as an ejector condenser for condensing engines in which the results obtained are more remarkable than when employed as an injector. The ejector condenser forces the water of condensation to heights due to the difference between atmospheric pressure and the vacuum in the condenser, and also ejects the whole volume of condensing and condensed water against atmospheric pressure, out of the necessarily rejected heat of the steam.

This is the total work ordinarily performed in working the pump to supply the water of condensation, and in working the air pump to preserve the vacuum. The whole of this work is performed by the ejector condenser without expenditure of useful energy by the engine. In other words the ejector converts a high pressure engine into a condensing engine without drawback, thereby increasing the efficiency of the engine through changing at once the temperature of its refrigerator from 212° to 175° or possibly to 150° F. Such a diminution of the temperature of the refrigerator adds notably to the efficiency of the engine as appears from the concluding paragraph of art. 19. As at present constructed, the energy due to the velocity with which the water reaches the ejector should not be neglected, and can be introduced into the preceding formulæ by introducing into (287), etc., instead of i_0 the expression

$$i_0 + \frac{v_0^2}{2g} \quad . \quad . \quad . \quad . \quad (291)$$

in which v_0 is the velocity of approach.

Two Pennsylvania gentlemen have returned from China, whither they went about a year ago to examine, for the Chinese Government, the oil grounds of the Island of Formosa. They report that a well was drilled through soapstone 396 feet; then 136 feet of drill pipe were put in and 265 feet of casing. No more casing could be got in owing to the caving in of the rock. At 348 feet depth a large vein of salt water was struck, and it was found impossible to go more than 48 feet deeper. Fifty barrels of oil, says the *Scientific American*, were pumped in ten days. The oil is very light in color and gravity, and was burned in lamps without refining. The property belongs to the Chinese Government.

THE TURBINE-WHEEL DISCUSSION.

A REVIEW OF THE REPLY OF PROFESSOR TROWBRIDGE.

By Prof. WM. H. BURR.

Written for Van Nostrand's Engineering Magazine.

As Professor Trowbridge refrains from giving any detailed proof regarding the main points of this discussion, he leaves them in essentially the same positions in which my April article placed them. Proof of a little more analytical character of what was therein shown will, however, be given in this article. Two or three points, also, of his last communication will be examined.

The superabundance of his general assertions can only, and simply, be met by other assertions equally general, equally futile, and, consequently, equally valueless.

Such assertions, therefore, are entitled to no consideration and will receive none.

He seems to the writer to raise a cloud of "confusion" with no apparent object, unless, perhaps, the purpose of discreetly taking refuge within it.

In view of the different statements in the two articles of Professor Trowbridge, it becomes a matter of no little interest to determine just what that is to which he objects, in "the theoretical investigations of the turbine wheel, as given by Rankine, Weisbach, Bresse and others."

In his March article occurs the following paragraph:—

"The theorem or axiom insisted upon by these writers was that the 'water must enter the wheel *without shock*, and hence, the mathematical condition that the *tangential velocity of the wheel, where it receives the water, and the corresponding component of the velocity of the entering water* must be equal, the effect of which is to prevent all *impulsive* effects of the entering water."

Does Professor Trowbridge mean by "*corresponding component*" a rectangular or oblique component? If he means the former then, truly, all impulsive effects of the entering water are prevented. But neither Weishbach nor Bresse *even state* that the rectangular component of the entering water, tangent to the wheel circumference, must be equal to the tangential velocity of the wheel; it is an oblique component which they make equal to that velocity.

So far as the formulæ of those writers are concerned the rectangular component of the entering water, tangent to the wheel circumference may be greater than, equal to, or less than, the tangential velocity of the wheel; and in the first of these three cases an impulsive effect of the water will be assured.

The objections of Professor Trowbridge do not, therefore, hold if his "corresponding component" means "rectangular component."

It may be supposed, therefore, that he objects to the oblique tangential component being made equal to the tangential velocity of the wheel. But these authors *advise* and "insist upon" this equality (without in any way affecting their formulae for best velocity and efficiency), in order that the water may enter the wheel tangentially to the bucket, and thus avoid the loss of energy due to what they call "shock," *i.e.*, impact of water in water.

It is important to notice in passing that this elimination of "shock" does not prevent "impulsive action" of the water.

From the paragraph quoted above it is seen that Professor Trowbridge objects to the "theorem or axiom" that "water must enter the wheel *without shock*." This taken in connection with what was said above in reference to the "oblique component," forces on to the conclusion that he objects to the water entering the wheel tangentially to the bucket. On the first page of his May article, however, he says: "I do not 'object' to the authors named insisting on the principle that the water must enter the wheel without 'shock.'"

Consequently he does not, after all, object to the water entering along the bucket, and therefore to the equality of the oblique tangential component and tangential wheel velocity.

What then does the quoted paragraph mean, if it means anything?

It is idle for him to make reference to the "*reaction* wheel," for it was shown above in a general way, and will be shown in detail hereafter, that impulsive action of the water is not prevented by such a condition of entrance.

In fact as a general principle the greatest impulsive effect will be obtained, other things being equal, by water passing on the surface, against which it impinges, in a tangential direction; since in that case all loss in eddies caused by impact of water in water will be avoided. This difference does not appear in the ordinary formulae, simply because the loss spoken of is omitted in them.

Since, therefore, Weisbach and Bresse do not in any way suppose the rectangular component of the velocity of the entering water to be equal to the tangential velocity of the wheel at the same point, and since in his May article Prof. Trowbridge states that he does not object to the water entering tangentially to the bucket, his objections against those authors are contradictory and futile.

He admits that there can be no objection against Rankine on the basis of what he does in reference to the oblique component of the entering water, or the bucket, for he (Rankine) deliberately ignores the whole of the bucket or blade, except that part of it which gives direction to the effluent water, and he thereby admits all I claimed for Rankine's formulae. I simply held the point that (omitting all loss caused by eddies consequent upon impact of water in water, or shock) the formulae of Rankine, as well as those of Weisbach and Bresse, are not dependent on the angle at which the bucket cuts the wheel circumference at the point of entrance.

I used the word "shock" in the same sense in which Bresse and Weisbach use it in the same connection, *i.e.*, as indicating the violent disturbance manifested by eddies, and not as indicating impulsive effect of the water. The two things are very different.

In order, however, to avoid all confusion on this point, the word "shock" will be used not to indicate impulsive effect, but, as heretofore, to indicate that "violent disturbance," as Bresse calls it, in which energy is lost by eddies, &c.

I readily agree with Prof. Trowbridge that it is necessary for Rankine to make $v=ar$ in order to get the greatest efficiency from his formulæ when friction is neglected. His method of investigation (which is considerably different from that of Weisbach and Bresse) renders such a substitution necessary.

But Professor Trowbridge has neglected to follow Rankine a little farther. In Art. 177 of his "Steam Engine," Rankine discusses the "efficiency of turbines, allowing for friction." He goes back, for the outward flow turbine, to the general expression for efficiency in Art. 174, introduces a proper term for the friction and then writes the whole expression in terms of $z=\frac{ar}{\sqrt{2gh_1}}$, in which h_1 is the total head. He then proceeds by the ordinary methods of the calculus to determine what value of z will make the expression for the efficiency a maximum. *He does not do it by making* $v=ar$, but in speaking of friction, states, "this cause of loss of work not only diminishes the efficiency of the turbine, but *diminishes very considerably the speed of greatest efficiency.*" The italics are my own.

Taking Professor Trowbridge, therefore, on his own ground, it is seen that his objections against Rankine's general formula for included friction for outward flow turbines do not exist.

Omitting friction, Rankine's general value of best efficiency takes either one of the following forms:—

$$\varepsilon=\frac{2}{2+n^2\tan^2\beta}=\frac{2}{2+\frac{\tan^2 a}{n^2}} \quad \ldots . (1)$$

a is the angle at which the guides cut the wheel circumference at the point of entrance into it, and β is the angle at which the buckets cut the wheel circumference at the point of exit from it. If β is zero, the efficiency is unity; as with the formula of Prof. Trowbridge. Eq.(1) will be used hereafter.

There are two methods of showing in detail that the formulæ of Weisbach and Bresse do not depend upon the angle at which the bucket cuts the wheel circumference at the point of entrance.

Proceeding with Rankine, all of the bucket except that part which gives direction to the effluent water may be ignored, or omitted entirely from consideration. Or the angle at which the bucket cuts the wheel circumference at the point of entrance may be given a different value from any that can possibly enter the desired formulæ.

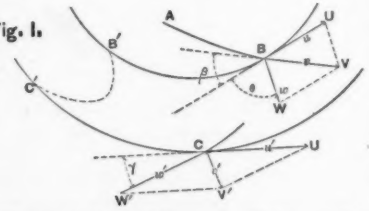
The first method will be used with the equations of Bresse and the second with those of Weisbach.

Before proceeding with Bresse's formulæ there is one point to be noticed.

In his May article, page 373, Professor Trowbridge states that his Fig. 1 (given in that page) is copied from Mahan's translation of Bresse; but it differs from that figure in one particular very important to this discussion. In his Fig. 1, which he states he copied, the angle between w and u is 90°, while in Mahan's Bresse (edition of 1869) that angle is about 101°, as nearly as can be measured. The importance of this point will be at once perceived, if it be remembered that Professor Trowbridge is endeavoring to show that the rectangular component of the entering velocity of the water is equal to the tangential velocity of the wheel, although Bresse lays down no such condition. Consequently the changed figure would apparently make Bresse indicate, through it, that which Professor Trowbridge, only, holds.

The true figure is the Fig. 1 of this article, with this exception: the end only of the bucket which directs the effluent water is retained; the rest of it is omitted.

Fig. 1.



The notation of Bresse will be used as seen in the figure. It is unnecessary to explain it here. As distinctly stated by Bresse, *all* losses of head are neglected except that due to the final absolute velocity v'.

The two following equations may then be written:

$$v^2 = 2g\left(H + h + \frac{p_a - p}{\pi}\right) \quad . \quad . \quad . \quad (2)$$

$$w'^2 - w^2 = 2g\left(h' + \frac{p - p'}{\pi}\right) + u'^2 - u^2 \quad . \quad (3)$$

The velocity w is simply the velocity relative to the wheel at point of entrance into it. Its magnitude and directions depend entirely upon the relative proportions and directions of u and v, or, in other words, upon u, v and β, and nothing else. In the figure θ is simply the angle which w makes with a tangent at B. Having thus determined w, w' is at once known in magnitude from (3). As is well known, the "fictitious" head $\frac{u'^2 - u^2}{2g}$ is entirely independent of the path described and dependent only upon the angular velocity and distances from the center. It is to be noticed that, thus far, w' is only given in magnitude; its direction is known as soon as γ is given; its magnitude does not in any way depend upon that angle.

A simple law of hydrostatics gives—

$$\frac{p_a}{\pi} + h + h' = \frac{p'}{\pi} \quad . \quad . \quad . \quad . \quad (4)$$

The law of continuity gives—

$$brv \sin \beta = b'r'w' \sin \gamma \quad . \quad . \quad (5)$$

The triangle in the figure gives—

$$w^2 = u^2 + v^2 - 2uv \cos \beta \quad : \quad . \quad (6)$$

$$v'^2 = u'^2 + w'^2 - 2u'w' \cos \gamma \quad . \quad (7)$$

Also evidently—

$$u'r = ur' \quad . \quad . \quad . \quad . \quad (8)$$

Eq. (6) might have been written immediately after eq. (2) but they are taken in the same order with Bresse. By supposing the angle γ small Bresse writes:

$$u' = w' \quad . \quad . \quad . \quad . \quad (9)$$

It is now important to notice that the equations (2) to (9), inclusive, have been determined without any regard to the bucket at B in Fig. 1; after having omitted the consideration of all resistances, they are in no sense dependent upon it. They hold whether that part of the bucket exists or not.

Now, the equations (2) to (9) are *the only ones* which Bresse uses in determining v'^2.

There is no necessity of here following through the operations by which he obtains that quantity; he simply combines

eqs. (2) to (9) and no other. Any one can verify this statement by turning to pages 85 and 86 of Mahon's Bresse. By the mentioned combination Bresse obtains—

$$v'^2=2gH\frac{b\tan\beta}{b'\sin\gamma}(1-\cos\gamma) \quad . \quad (10)$$

He then goes on to treat of some other matters, after which, page 89, he says:

"Again it might be proposed, for a turbine known to be working with a maximum effective delivery, to seek this delivery as well as the dynamic effect. As we are supposing that all losses of head, other than that due to the velocity of exit v', may be disregarded, the head that is turned to account will be

$$H-\frac{v'^2}{2g}$$

and consequently the productive force μ will be expressed by

$$\mu=1-\frac{v'^2}{2gH}" \quad . \quad . \quad . \quad (11)$$

It is seen therefore that μ, or the efficiency, in eq. (11) is written by Bresse at once; without any regard to any preceding equation whatever. Hence it does not depend upon the angle at which the bucket cuts the circumference at the point of entrance into it, directly or indirectly, implicitly or explicitly, or in any other manner known to the human understanding. Whenever, therefore, Prof. Trowbridge states that it is so dependent, he is simply in error. He is also in error when he states in his last article, page 374, that w depends on the bucket angle at B, Fig. 1. For eq. (6) shows that w depends only on u, v and β, and that bucket angle may have any value whatever (or that part of the bucket may not exist at all, so far as the equations are concerned) while those quantities remain the same. w is, therefore, entirely independent of the angle in question.

Now it has been assumed that there is no loss of energy by "shock," but such a loss would exist if the buckets, at the point of entrance, made an angle different from that of the relative component w, hence, in an actual case, in order to get the best results, that angle should be the same with that of w, for then the water will glide along the bucket without disturbance. Let θ be this angle, then from Fig. 1 there is easily deduced:

$$\frac{u}{v_1}=\frac{\sin(\theta+\beta)}{\sin\theta} \quad . \quad . \quad . \quad . \quad (12)$$

Bresse writes this equation between those two which I have numbered (8) and (9), but I have omitted it until the present in order to make prominent the fact that *it has absolutely no connection whatever with any of the equations from which v'^2 is deduced;* and it evidently has no connection with equation (11). There is no need of bidding its influence "out," for it is never "in." Bresse writes it, it is true, and states in connection with it that the water must enter tangentially for the best results (just what I explicitly stated in my previous paper), but he does not combine it with any equation which precedes or follows it for the determination of v'^2 or the efficiency. This statement may also be verified by consulting Bresse.

The statement of Prof. Trowbridge, therefore, to the effect that Bresse combines eq. (12) with others to obtain v'^2 and the efficiency is, to put it very mildly, somewhat extraordinary.

He simply falls into the error of supposing that the supplement of the angle which w makes with u is necessarily the angle which the bucket makes with the tangent at the same point, whereas so far as the equations are concerned, there is no connection between them; the former is fixed when u, v and β are known, but the latter may have any value whatever. Again it is important to observe that the equations are based on no particular relation between the rectangular tangential component of v, and u; much less is equality any where assumed. So far as the equations are concerned, that component of v may be greater or less than, or equal to, u. The formulae are then perfectly general in this respect.

Introducing v'^2 from eq. (10), in eq. (11), there results:—

$$\mu=1-\frac{b\tan\beta}{b'\sin\gamma}(1-\cos\gamma) \quad . \quad (13)$$

Eq. (13) will be used again.

The preceding results might also have been shown by taking any bucket angle different from θ, and then by establishing the equations without any regard to it; this method will now be applied to Weisbach's equations.

Fig. 2 is the figure to be used; it is

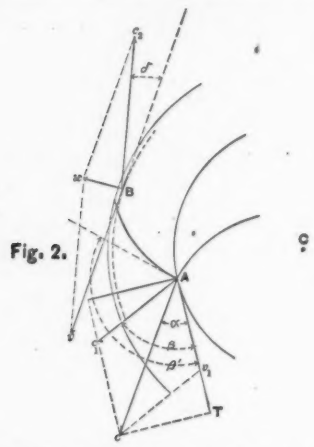


Fig. 2.

very similar to the one found in Art. 250 of Du Bois' Weisbach. The notation is the same as that of Weisbach (and does not need to be explained here) with this exception: β' will represent the angle made by the bucket with the circumference of the wheel at the point of entrance into it, while β *represents the angle which the relative component* c_1 *makes the circumference at the same point.* This distinction is important.

From the triangle c A v_1 Weisbach (Art. 250) writes,

$$c_1^2 = c^2 + v_1^2 - 2cv_1 \cos a \quad \ldots . \quad (14)$$

By the aid of this equation, the fictitious head $\frac{v^2 - v_1^2}{2g}$ (see remarks following eq. (3) and the actual heads h_1 and h_2, he then writes,

$$\frac{c_2^2}{2g} = h_1 - h_2 - (1+\zeta)\frac{c^2}{2g} + \frac{c^2}{2g} + \frac{v^2}{2g} - \frac{2cv_1 \cos a}{2g} \quad \ldots . \quad (15)$$

Writing h for $h_1 - h_2$ and adding a term $\frac{\zeta_1 c_2^2}{2g}$, to represent the loss of head by friction in the wheel, there results from (15),

$$(1+\zeta_1)c_2^2 = 2gh + v^2 - 2cv_1 \cos a - \zeta c^2 \quad . \quad (16)$$

With Weisbach let Q be the discharge from the wheel; F the sum of all exit orifices from the guide curves, and F_2 the sum of the exit orifices from the wheel. Then by the law of continuity,

$$Q = Fc = F_2 c_2 \quad \therefore \quad c = \frac{F_2}{F} c_2;$$

also, evidently,

$$v_1 = \frac{r_1}{r} v.$$

Assuming that δ is small Weisbach takes $v = c_2$.

Making the substitutions in eq. (16);

$$v = \sqrt{\frac{2gh}{2\frac{F_2}{F}\frac{r_1}{r}\cos a + \zeta\left(\frac{F_2}{F}\right)^2 + \zeta_1}} \quad \ldots . \quad (17)$$

This is Weisbach's expression for best velocity and is found in Art. 251 of Du Bois' translation. It will be observed that $v = c_2$ is the same condition that Bresse uses.

From Fig. 2 there are easily deduced the following relations:

$$\frac{F_2}{F} = \frac{r \sin \delta}{r_1 \sin a} = \frac{r_1 \sin \beta}{r \sin(\beta - a)}$$

Introducing these conditions successively in (17) there result the two values of v_1:

$$v_1 = \frac{r_1}{r} v = \sqrt{\frac{2gh}{2\left(\frac{r}{r_1}\right)^2 \frac{\sin\delta}{\sin a}\cos a + \zeta\left(\frac{r}{r_1}\right)^4 \left(\frac{\sin\delta}{\sin a}\right)^2 + \zeta_1\left(\frac{r}{r_1}\right)^2}} \quad \ldots . \quad (18)$$

$$v_1 = \frac{r_1}{r} v = \sqrt{\frac{2gh}{2\frac{\sin\beta\cos a}{\sin(\beta - a)} + \zeta\left(\frac{\sin\beta}{\sin(\beta - a)}\right)^2 + \zeta_1\left(\frac{r}{r_1}\right)^2}} \quad \ldots . \quad (19)$$

It is seen, now, that these two values of v_1 are identical and entirely independent of β', in fact would hold if that end of the bucket did not exist. The only condition involved is simply $v = c_2$. In applying therefore eq. (19) to an actual case, β *should not be taken as the angle which the bucket makes with the circumference of the wheel at the point of entrance into it,* but should be calculated by the simple formula for a triangle, knowing c, v_1 and a; or it may be determined from the equation which immediately precedes eq. (18).

Here I will state what, by an oversight, I omitted to state in my April article.

In that article the values for v_1 were calculated from eq. (19) taking β directly from the wheel. This I did to save the

labor of estimating them as directed above, knowing, since they are excellent examples, that the values would differ little from estimated ones. In the Boott Center-Vent and Westerly Swain wheels (my 3d example) the differences were a little greater than I expected, but not enough to make any essential difference in the results.

In the Center-Vent Boott wheel, for instance, v_1 by eq. (18) becomes

$$0.758\sqrt{2gh}=22.2 \text{ feet.}$$

But experiment 27 of the same series, from which I took my example, gives a little higher efficiency than experiment 30, which I took, and v_1 in that experiment was found to be 20.8 feet. The agreement is therefore as close as before. In the Westerly Swain wheel (my third example) eq. (18) gives $v_1=20.78$ feet, while experiment gave $v_1=21.48$ feet; an agreement considerably closer than before.

I explicitly stated the value of the empirical co-efficient I used as 0.075. In art. 260 of DuBois' translation Weisbach states where Professor Trowbridge may find an account of the experiments by which the values "$\zeta=\zeta_1=0.05$ to 0.10" were fixed.

In art. 260 of DuBois' translation is found Weisbach's expression for efficiency. Putting L_1 for the work performed and, as before, h for the total fall, there may at once be written;

$$L_1=\left(h-\frac{\zeta c^2+\zeta_1 c_2^2+w^2}{2g}\right)Q\gamma \quad . \quad (20)$$

Using the relations between c, c_2 and v, already given, there results for the efficiency, after taking v from eq. (18);

$$\eta=1-\frac{\zeta\left\{1+\left(\frac{r\sin\delta}{r_1\sin a}\right)\right\}+4\left(\sin\frac{\delta}{2}\right)^2}{\zeta\left\{1+\left(\frac{r\sin\delta}{r_1\sin a}\right)^2\right\}+2\cot a\sin\delta} \quad (21)$$

Eq. (21) will be recognized at once as the one used in my previous communication.

It will now be at once perceived that eqs. (18) or (19) and (21) are entirely independent of the angle β'; it is not possible for it to affect them. If possible, it is still more absurd to suppose that either (18) or (19), or (21) is based on the assumption that the tangential rectangular component of c, Fig. 2, is equal to v_1. The figure was purposely so drawn as to represent that component as greater than v_1, as does Weisbach, in which case the water would have an impulsive effect on the wheel depending upon the excess v_1 T.

Weisbach's formulæ for best velocity and efficiency, therefore, are *not* based either upon the "axiom" that the water should enter tangentially to the bucket, or upon the axiom that the water shall not act by impulse.

They are based upon the condition $c_2=v$, but that is all. They do not involve any term which represents the loss by shock any more than they involve terms which represent loss arising from appreciable thickness of buckets and guide curves, or fluid friction in the flume or penstock, and any objection which is brought against the formulae on the basis of any one of these omissions holds equally on the basis of the omission of all the others.

The pseudo-reasoning of Professor Trowbridge, in his last article in the vicinity of his Fig. 2, is too deeply involved in hopeless absurdity to be worthy of serious attention.

In his last article, page 375, Professor Trowbridge states essentially that the formulæ of Weisbach and Bresse do not indicate that a small value of a (Weisbach's notation) or β (Bresse's notation) should be taken. In my last article I showed that Weisbach's formula for efficiency calls for a small value of a, and took occasion to criticise his advice in regard to making it unnecessarily large. Bresse's eq. (13) shows that the smaller is the angle β the greater is the efficiency; and Rankine's eq. (1) shows the same thing (his a is the same as Bresse's β). The small guide curve angles of the Lowell wheels are, therefore, nothing more than those which the formulae of Weisbach, Bresse and Rankine indicate ought to exist. It is nothing to the point to quote the *opinions* of those authors, for in his first article (see its heading) Professor Trowbridge set out to show "the inapplicability of the theoretical investigations of the turbine wheel as given by Rankine, Weisbach, Bresse and others." It is a matter of little consequence what their opinions were.

In Art. 262, however, Wiesbach advises that *a be made from* 20° *to* 30°. In the same article he advises that the exit angle of the wheel be less than 20°.

In his last article Professor Trowbridge states that the "best modern wheels, introduced by Boyden and Francis . . . . . are *impulse*-and-reaction wheels," and, in the paragraph following my quotation, he speaks in particular of the inward flow turbines. He thus states that the water acts in those wheels partly by impulse. The experiments of Francis and Mills, however, show the contrary. Figs. 3, 4, 5 and 6 are the velocity diagrams belonging to the four examples given in my April article; in fact, Figs. 3 and 4 are reproduced from that article.

The notation is that of Weisbach, c being the exit velocity from the guides and v_1 the tangential wheel velocity at the same point.

Fig. 3.

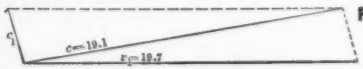


Fig. 4.

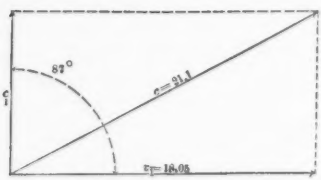


Fig. 5.

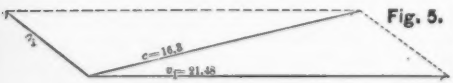
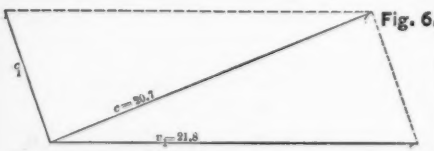


Fig. 6.



The velocities c and v_1 are both experimental in the four cases, and may be easily verified by any one taking the trouble to look up the data given in my previous article. Figs. 3, 5 and 6 belong to inward flow turbines, and in each of those cases, since the component of c in the direction of v_1 is less than v_1, *it is impossible for the water to do any work by impulse*; that is, they are simply reaction wheels.

Fig. 4 belongs to the Tremont wheel. In that case, since the angle between c_1 and v_1 is only 87°, there is a very little work done by impulse; but since that little is due to 3° only, it is essentially none.

If Prof. Trowbridge will read my previous article with a little more care, he will see that what I called an "admirable arrangement" was simply the means adopted to prevent leakage in the Lowell wheels, which I took for examples. Also that I did not speak favorably of the idea of Bresse and Weisbach of so proportioning α and β, as to prevent unbalanced pressure outward between wheel and guides.

Perhaps the most remarkable characteristic of Prof. Trowbridge's last article is his claim to originality in the investigation of his first paper. He has given, he says, the theory of impulse-and-reaction wheels "for the first time"!!!

Let any one turn to Fig. 1 of his March article and the page following of his text. The general expression for work which he deduces is:

$$W = M(xu + v_r \cos \gamma . u) \quad . \quad . \quad (22)$$

From his figure $v_r = \sqrt{v_1^2 + u^2 2uv_1 - \cos\alpha}$ and $x = v_1 \cos \alpha - u$; these values in eq. (22) give:

$$W = M (v_1 \cos\alpha + \sqrt{v_1^2 + u^2 - 2v_1 u \cos\alpha} \cos\gamma - u)u \quad (23)$$

Now let any one turn to Art. 231 of Du Bois' translation, where Wiesbach gives the theory of the turbines of Borda and Bondin. About the middle of page 345, Weisbach has the formula.

$$L = \frac{Q\gamma}{g} (c\cos a + \sqrt{c^2 + v^2 - 2cv \cos a}.\cos\delta - v)v \quad . \quad (24)$$

In eq. (24) L, $\frac{Q\gamma}{g}$, c, v and δ are exactly the same quantities as W, M, v_1, u and γ, respectively, in eq. (23). Consequently, the equation of Professor Trowbridge is identical with that of Weisbach.

Weisbach then makes $\cos\delta = 1$; Prof. Trowbridge does the same with $\cos\gamma$. Weisbach next seeks by the method of the calculus what value of v will make L a maximum and finds $v = \frac{c}{2\cos a}$; Professor Trowbridge does the same with W in respect to u and finds

$$u=v_r\left(\text{Weisbach's } \frac{c}{2\cos a}\right).$$

Finally, Weisbach puts $v=\frac{1}{2}\frac{c}{\cos a}$ in his L and finds an efficiency of unity; with great fidelity Professor Trowbridge puts $u=v_r$ in his W and also finds an efficiency of unity. The identity of the formula and operations of Weisbach and Professor Trowbridge and the great degree of similarity in the figures they use are quite remarkable, and render any comment superfluous.

A NEW CENTER OF GRAVITY FORMULA OF GENERAL APPLICABILITY.

By J. WOODBRIDGE DAVIS, C.E.

Written for VAN NOSTRAND'S MAGAZINE.

The formula

$$\frac{1}{2}D+\frac{(B-A)D^2}{12V}, \qquad (1)$$

wherein A is the area of the generatrix of a space, in its initial position, B_1 its area in its final position, D is the distance between these positions, and V is the amount of space between, is an expression representing a distance, or linear measurement, like in kind to D, because the second term is a product of four dimensions divided by a product of three dimensions of like kind. If the space be a plane, the second term is still, as well as the first, an expression of a single dimension. This expression has been found to represent the distance of the center of magnitude from the initial end, of each of a number of shapes, or the center of gravity of a mass of uniform density, which fills the space.*

To determine the extent of its applicability, let us first consider two spaces,

$$y=f_1(x)_0^D, \qquad y=f_2(x)_0^D. \qquad (2)$$

Let A_1, B_1, be the end areas of first, V_1, its volume, D, the distance between these limits, and D_1, the distance, from first end, of the center of its magnitude. Let A_2, B_2, V_2, D, D_2, be the similar values of the second space, both lying between same limiting planes, as indicated in (2).

If formula (1) apply to each of these spaces, then

$$D_1=\frac{1}{2}D+\frac{(B_1-A_1)D^2}{12V_1},$$

$$D_2=\frac{1}{2}D+\frac{(B_2-A_2)D^2}{12V_2}.$$

The distance, D_3, of the center of gravity of the two spaces from the initial plane, is

$$D_3=\frac{D_1V_1+D_2V_2}{V_1+V_2}=$$

$$\frac{\frac{1}{2}DV_1+\frac{1}{12}(B_1-A_1)D^2+\frac{1}{2}DV_2+\frac{1}{12}(B_2-A_2)D^2}{V_1+V_2}$$

$$=\frac{1}{2}D+\frac{(B_1+B_2-A_1-A_2)D^2}{12(V_1+V_2)}=\frac{1}{2}D+\frac{(B_3-A_3)D^2}{12V_3}$$

wherein A_3, B_3, V_3, are the areas of ends, and volume of combined space. Therefore, formula (1) applies to the space

$$y=[f_1(x)+f_2(x)]_0^D=f_3(x)_0^D.$$

In the same manner it may be shown that the formula applies to the space

$$y=[f_3(x)+f_4(x)]_0^D,$$

if it apply to space, $y=f_4(x)_0$, and so on. Consequently, *if formula* (1) *apply to each of any number of spaces between same limits, it applies to their sum.*

From a similar course of reasoning it follows that *If formula* (1) *apply to all but one of any number of spaces between same limits, it does not apply to their sum.* Also, *if the formula apply to certain spaces between same limits and do not apply to certain other spaces, more than one, between the same limits as before, the formula does not apply to the sum of all the spaces, except in special cases, when its error for some of the spaces is balanced by its error, with opposite sign, for the rest.*

Now let us consider the expression for

* Formulæ for R. R. Earthwork, Second Edition, p. 105

all spaces, $y=F(x)$.* Each term of this is of the form Kx^n; and the distance of the center of gravity, from the first end, of the space represented by $y=Kx^n$, is

$$\frac{\int_0^D Kx^{n+1}dx}{\int_0^D Kx^n\,dx}=\frac{n+1}{n+2}D.$$

Formula (1) applies to this space when

$$\tfrac{1}{2}D+\frac{(KD^n-K(0)^n)D^2}{\frac{12}{n+1}KD^{n+1}}-\frac{n+1}{n+2}D=0. \quad (3)$$

This is true when $n=0$. For all other values of n, (3) becomes

$$\tfrac{1}{2}D+\frac{n+1}{12}D-\frac{n+1}{n+2}D=0. \quad (4)$$

From this $n^2-3n+2=0$;

whence $n=\frac{3}{2}\pm\frac{1}{2}=2$ or 1.

It follows from this and the rules written in italics, that the center of gravity formula applies, between any limits, to those spaces only which are represented by the equation

$$y=a+bx+cx^2; \quad (5)$$

and it applies to special cases only, [*i. e.*, between special limits,] of all other spaces.

By comparison of (5) with similar equation, [eq. (11), p. 412, May No. of this Magazine,] representing the limit of the prismoidal formula's applicability, which equation is

$$y=a+bx+cx^2+dx^3, \quad (6)$$

it is seen that the center of gravity formula is, *practically*, co-extensive with the prismoidal formula; because the generatrices of very few, if any, practical shapes vary as cubic functions of the path. This formula will, therefore, serve in the same way as the prismoidal, as a widely general rule, which renders unnecessary the demonstration, recollection and use of a large number of special rules.

Thus, the chapter on center of gravity in a treatise on mechanics can be much abbreviated by use of this formula. The fact that the generatrix of a space, expressed in terms of its distance from initial end of that space, or from any position whatever, since to shift the axis of Y does not change the degree of eq. (5), is all that requires demonstration. The position of the center of magnitude in that space is, then, immediately indicated by formula (1), which may be reduced for particular cases.

For instances: Because the areas of generatrices of triangle, pyramid, cone, and paraboloid, in terms of distance from their vertices, are, respectively, ax, $b(cx)^2$, $\pi(ex)^2$ and πfx, formula (1) applies to all of these and to all of their frusta. Therefore, for triangle, the distance of center of gravity from vertex is

$$\tfrac{1}{2}D+\frac{(B-0)D^2}{12(\frac{1}{2}DB)}=\tfrac{2}{3}D.$$

For trapezoid;

$$\text{dist. c. of g.}=\tfrac{1}{2}D+\frac{(B-A)D^2}{12(B+A)\frac{1}{2}D}=\frac{(2B-A)D}{3(B+A)}.$$

For pyramid or cone when B is area of base,

$$\text{dist. c. of g.}=\tfrac{1}{2}D+\frac{BD^2}{12(\frac{1}{3}BD)}=\tfrac{3}{4}D.$$

For frustum of this:

$$\text{dist. c. of g.}=\tfrac{1}{2}D+\frac{(B-A)D^2}{12(A+\sqrt{AB}+B)\frac{1}{3}D}=$$

$$\tfrac{1}{2}D+\frac{(B-A)D}{4(A+\sqrt{AB}+B)}.$$

The last result, which is formula (1) only changed by cancelation of the common factor D, is at once in the simplest form. Dr. Weisbach requires a page and a quarter of laborious demonstration to reach this result. See pp. 233-4, Eckley Coxe's edition of *Theoretical Mechanics*.

For paraboloid, the radius of whose base is r,

$$\text{dist. c. of g.}=\tfrac{1}{2}D+\frac{\pi r^2D^2}{12\pi r^2\frac{1}{2}D}=\tfrac{2}{3}D.$$

For frustum of this we obtain an expression similar to that for trapezoid, since its generatrix varies according to a function exactly similar. It is

$$\text{dist. c. of g.}=\tfrac{1}{2}D+\frac{\pi(r_2^2-r_1^2)D^2}{12\pi(r_2^2+r_1^2)\frac{1}{2}D}=$$

$$\frac{(2r_2^2-r_1^2)D}{3(r_2^2+r_1^2)}$$

Formula (1) applies to the sphere, because, with center as origin, the magnitude of generating circle varies as

* See introduction to article entitled *Prismoidal Formula* in May number of this Magazine.

$$\pi(r^2-x^2).$$

Hence, for hemisphere, measured from center,

$$\text{dist. c. of g.} = \tfrac{1}{2}r - \frac{\pi r^4}{12(\frac{2}{3}\pi r^3)} = \tfrac{3}{8}r.$$

Because the generatrix of the complete rectangular prismoid [Fig. 2, p. 414, May No. of this Magazine] varies as

$$a+bx+cx^2,$$

formula (1) applies to all rectangular prismoids and wedges, in direction perpendicular to bases.

$$\therefore \text{dist. c. of g.} = \tfrac{1}{2}D + \frac{(B-A)D^2}{12(A+4M+B)\frac{1}{8}D} =$$

$$\tfrac{1}{2}D\left\{1+\frac{B-A}{A+4M+B}\right\} = \frac{(B+2M)D}{A+4M+B}.$$

For wedge, measured from edge;

$$\text{dist. c. of g.} = \tfrac{1}{2}D\left\{1+\frac{B}{B+4M}\right\}$$

$$= \frac{(B+2M)D}{B+4M}.$$

To reach results similar to these for wedge and prismoid requires more than a page of Weisbach's Mechanics. By calculus, the demonstration is nearly as long.

In a manner similar and quite as easy it may be shown that formula (1) applies to any segment of common parabola, in direction perpendicular to axis, to all parallelograms, prisms, cylinders, prismoids, cylindroids, spheroids, hyperboloids, all segments of these between parallel planes, and to the class of shapes whose lateral boundaries are right line surfaces the type of which is illustrated in Fig. 6, p. 418, of May No. of this Magazine. The last class includes alone, columns, chimneys, piers, abutments, warped-faced wing-walls, banks, retaining walls, dams, earthwork solids, and numerous other practical shapes.

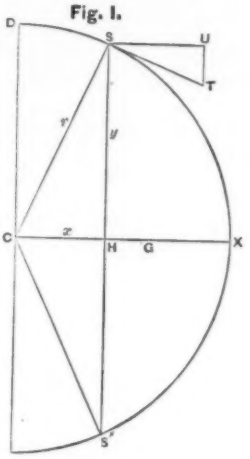
If the extremity, S, of the generatrix of a circle, Fig. 1, generate a length, dx, of the circumference in an infinitesimal time, dt, while it passes through center, then, at any distance, x, it will in same time generate a length equal to

$$dx.\frac{ST}{SU} = dx.\frac{r}{y} = dx.\frac{r}{\sqrt{r^2-x^2}}.$$

Hence formula (1) does not apply to arcs of circles.

But, if the circumference be revolved about CX, then the circumference of the generating circle whose radius is y, generates in the time, dt,

Fig. 1.



$$dx.\frac{r}{y}.2\pi y = 2\pi r dx,$$

which is the form of eq. (5), when $b=o$, $c=o$. Hence formula (1) applies to any zone of sphere, and the position of center of magnitude is, when h is altitude of zone,

$$\tfrac{1}{2}h + \frac{(2\pi r dx - 2\pi r\, dx)h^3}{12\ V} = \tfrac{1}{2}h.$$

Because formula (1) applies to the cone CSS′, and to the segment SXS′, as has already been shown, it may be used to determine the positions of the centers of gravity of these; and the c. of g. of sector may then be found by composition of moments.

Formula (1) applies directly to the spherical sector when D is understood to be the distance passed over by the center of magnitude of generatrix, a varying, concentric zone, and V to be the amount of space described by a plane generatrix, varying as the same function of, and passing over, the same path.

This is true, because the distribution of magnitude along the path D is identical for both modes of generation, and because, since formula (1) applies to one of these shapes, the variation of generatrix being as cx^2, it also applies to the other.

Thus is obtained for sector of sphere,

$$\text{dist. c. of g.} = \tfrac{1}{2}d + \frac{\text{B}d^3}{12(\frac{1}{3}\text{B}d)} = \tfrac{3}{4}d = \tfrac{3}{4}(r - \tfrac{1}{2}h) \quad . \quad . \quad (7)$$

where B is area of spherical circle, d is distance of its c. of g. from center of sphere, h is its altitude, and r is radius of sphere.

In this sense formula (1) applies to frusta of spherical sectors, [*i. e.*, between concentric zones], whether the zones be terminal or annular; also, in direction of sector's axis, to all divisions of these included each by two planes passed through axis; to circular sectors between any limits, to all portions of regular polygons and polyhedrons, which can be inscribed in circular and spherical sectors, respectively, between any limits, and to every shape, each part of whose generatrix varies as the same function of its distance from initial end.

Thus, it applies to the square diagonally, when the initial end is one vertex, and the generatrix is the broken line composed of the opposite two sides.

Applied in this way, formula (1) generally requires, preliminarily, the aid of some other method to determine the c. of g. of generatrix itself. In case of spherical sector, it applies both to the generatrix and to the space described thereby.

It will be observed from the foregoing that the only familiar simple shapes, such as are made examples in works on mechanics and engineering, to which formula (1) does not apply *directly*, are the circular arc, sector, segment and spandrel; and to latter three of these it applies either indirectly or with aid of composition of moments.

Formula (1) applies to all lines for which

$$\sqrt{\frac{dy^2}{dx^2} + 1} = a + bx + cx^2, \qquad (8)$$

where a, b, c, are arbitrary. If $c = o$, dy is always in integrable shape; but the integration is tedious. Eq. (8), however, shows at once that the only familiar line to which formula (1) applies is the straight line. To obtain another example, without much labor in the integration, make a and c zero and square, subtract unity, extract square root, and multiply by dx, each member of eq. (8). By integration the equation of a line, subject to formula (1) is found to be

$$y = \tfrac{1}{2}bx\sqrt{x^2 - \frac{1}{b^2}} - \frac{1}{2b}\text{ nap. log.}\left\{x + \sqrt{x^2 - \frac{1}{b^2}}\right\} + \text{const.}$$

Formula (1) applies in direction of axis to all surfaces formed by revolution of the lines for which

$$y\sqrt{\frac{dy^2}{dx^2} + 1} = a + bx + cx^2, \qquad (9)$$

where a, b, c, are arbitrary. The only familiar surfaces of revolution for which eq. (9) is true are the surfaces of cylinder, cone and sphere.

Formula (1) applies, as a center of pressure formula, to the common cases of hydrostatic pressure. The equation of any immersed plane figure, referred to intersection of its plane with surface of fluid, as the axis of Y, and to any line in its plane, perpendicular to this intersection, as the axis of X, is

$$y = \text{F}(x).$$

If θ be the angle of deviation of its plane from plane of fluid surface, and γ be the specific gravity of the fluid, supposed to be uniform in density, the pressure on the figure at a distance x is

$$y' = \text{F}(x)\gamma \sin\theta\, x = \text{F}'(x).$$

In order that formula (1) shall apply,

$$\text{F}'(x) = a + bx + cx^2.$$

$$\therefore \text{F}(x) = \frac{a + bx + cx^2}{\gamma \sin\theta . x} = \frac{a'}{x} + b' + c'x. \qquad (10)$$

Fortunately, the last member of eq. (10), when we make $a' = o$, represents nearly all the usual submerged plane shapes, since these are seldom other than the rectangle, triangle and trapezoid, with bases parallel to fluid surface. Because the pressure at every point of submerged surface is normal thereto, and equal in intensity to $\gamma \sin\theta\, x$, it may be represented by a line perpendicular to surface and varying in length as the same constant multiple of x. Therefore, the equation of entire pressure,

$$y = (b' + c'x)\gamma \sin\theta . x = bx + cx^2,$$

is also the equation of a space, whose generatrix is a rectangle varying as y in last equation, its sides remaining constant in direction. This shape can, consequently, be only a frustum of the rectangular pyramid, prism or prismoid.

We may, therefore, confine our attention wholly to these shapes, because the resultant of pressure passes through the center of magnitude of each.

Of course, as shown by expression (6), the prismoidal formula also applies to these shapes, and, in consequence, to the pressure. Indeed, it would seem that this is the simplest method of determining the total pressure. The following is the

RULE.

To find the total pressure on a submerged rectangle, triangle or trapezoid, whose bases are parallel to fluid surface, *Multiply top-width of shape by its distance beneath surface; call this* A. *Multiply bottom width by its distance beneath surface; call this* B. *Multiply mid-width by its distance beneath surface; call this* M. *Then total pressure is* $\frac{1}{6}$ *of distance between end-widths, multiplied by the specific gravity of fluid, and again by*

$$A+4\,M+B.$$

When top-width is w_1, and its distance beneath surface is h_1; and w_2, h_2, correspond to lower width, while D is distance between, the formula is

$$P=\tfrac{1}{6}D\gamma[w_1h_1+(w_1+w_2)\,(h_1+h_2)+w_2h_2] \quad . \quad . \quad (11)$$

If it be desired to determine the component of pressure in any direction, use, instead of D, the projection of D normal to that direction.

The formula determining the distance of center of pressure, measured from fluid surface along submerged plane, is

$$d_1+\tfrac{1}{2}D+\frac{(B-A)D^2\gamma}{12P}, \quad . \quad . \quad (12)$$

where d_1 is distance from fluid surface to top base, measured on plane of figure, and the other symbols are as before. (12) may be written

$$d_1+\tfrac{1}{2}D+\frac{(B-A)D}{2(B+4M+A)}, \quad (13)$$

or

$$d_1+\tfrac{1}{2}D+\frac{(w_2h_2-w_1h_1)D}{2[w_1h_1+(w_1+w_2)(h_1+h_2)+w_2h_2]} \quad . \quad . \quad . \quad (13a)$$

When, instead of h_1, h_2, we know the distances d_1, d_2, along submerged plane, formula (11) becomes

$$P=\tfrac{1}{6}D\gamma\sin\theta[w_1d_1+(w_1+w_2)(d_1+d_2)+w_2d_2] \quad . \quad . \quad . \quad (14)$$

and formula (13a) becomes

$$d_1+\tfrac{1}{2}D+\frac{(w_2d_2-w_1d_1)D}{2[w_1d_1+(w_1+w_2)(d_1+d_2)+w_2d_2]} \quad (15)$$

as simple as before.

To find distance of center of pressure below fluid surface, multiply (15) by $\sin\theta$, or use in (15), instead of d_1, d_2, D, their vertical projections, if these be known

EXAMPLES.

The submerged figure is a triangle whose base is at fluid surface. Here, d_1 and w_2 are zero; consequently, the distance of center of pressure is $\frac{1}{2}$D. The pressure is

$$\tfrac{1}{6}D\gamma\sin\theta\,w_1d_2 \text{ or } \tfrac{1}{6}D\,\gamma\,w_1h_2.$$

This corresponds to the case of the entire middle branch of the complete prismoid, Fig. 2, p. 414, May No. of this Magazine.

The vertex of the triangle is at surface, and the base is parallel thereto. Here w_1 and d_1 are zero; and, in consequence, formula (15) reduces to $\frac{2}{3}$ D. Then,

$$P=\tfrac{1}{3}D\gamma\sin\theta.\,w_2d_2, \text{ or } \tfrac{1}{3}D\gamma\,w_2h_2.$$

The figure is a rectangle. Here $w_1=w_2$, and the formula (15) becomes

$$d_1+\tfrac{1}{2}D+\frac{(d_2-d_1)D}{6(d_2+d_1)}.$$

$$P=\tfrac{1}{2}D\gamma\sin\theta.\,w_1(d_1+d_2).$$

The distribution of pressure on submerged trapezoids corresponds to the distribution of magnitude in the various segments of the complete rectangular prismoid, as wedges, etc.

If the plane of submerged figure be parallel to fluid surface, formula (13a) shows that the center of pressure is coincident with the center of magnitude of the plane shape itself. When, now, $w_1=w_2$, we have the case corresponding to that of the rectangular prism.

While defining the rectangular prismoid on p. 414, May number of this magazine, as a shape generated by a moving rectangle, the product and quotient of whose two dimensions vary, we noticed two other shapes, in one of which—the pyramid—the product is variable and the quotient constant, and in the other of which—the prism—both are constant; also, a fourth shape was de-

scribed for which the product is constant and the quotient variable. All these cases occur in hydrostatics. The only one not mentioned is that of the figure whose generatrix varies in magnitude as $\frac{a'}{x}$, the first term of last member of eq. (10), while the pressure varies as $a'x$.

Formula (13), or its equivalent, formula (15), in same manner as formula (1), saves the calculator the inconvenience of remembering numerous rules, because the reduction for special cases, a few of which have been illustrated, can be instantly effected before he proceeds to the numerical part of the work. The same may be said of the prismoidal formula, as applied to the summation of these pressures.

The only other submerged figure mentioned in treatises on hydrostatics is the circle, which is usually the surface of a valve. Formula (15) does not apply to this. If r be its radius, and h the depth of centre below surface, then

$$P = h\pi r^2\gamma;$$

and dist. cen. of press. $= h + \frac{r^2}{4h}$.

The statical moment of a material shape, represented by eq. (5), referred to an axis parallel to plane of generatrix, and at a distance d_1 beyond its initial position, is the product of V by the sum of d_1 and expression (1).

$$\therefore \text{Statical moment} = (d_1 + \tfrac{1}{2}D)V + \tfrac{1}{12}D^2(B-A). \qquad (16)$$

This has exactly same advantages as (1).

Formula (16) is very convenient when we would determine the center of gravity, or the statical moment of a compound figure, every part of which is a shape represented by eq. (5). The first term of second member shows that the *c.* of *g.* of each part may be assumed to be half way between its ends; whereafter, by composition, very easily a false statical moment of the whole figure can be obtained. This should be corrected by the second term of second member applied to each part.

The statical moment of the pressure, whose resultant occupies the position indicated by (15), is, referred to intersection of submerged plane and fluid surface, the product of P and expression (15).

To find the moment of the horizontal component of P, referred to fluid surface, which is the moment usually required, multiply the former moment by $(\sin\theta)^2$.

This is useful when we would find the position of center of pressure of a compound figure. Such a figure may be divided into triangles, trapezoids and rectangles, whose bases are parallel to fluid surface; and the moment of each may be found as above.

The moment which is most often required is that of the horizontal component of P, referred to the lower base. This is the product of $\sin\theta$. P and $\sin\theta$. $[d_2 - (15)]$, which is

$$\tfrac{1}{2}D\sin^2\theta.P - \tfrac{1}{12}D^2\gamma\sin^3\theta.(w_2d_2 - w_1d_1), \quad \text{or}$$
$$\tfrac{1}{12}D^2\sin^3\theta.\gamma[2w_1d_1 + (w_1 + w_2)(d_1 + d_2)]. \qquad (17)$$

This is the moment which tends to overturn the solid whose surface receives the pressure.

Formula (1) will prove useful to the practical engineer, since few shapes to which it is inapplicable come under his consideration. In his service it will be especially simple, for the reason that he will be very likely to have already calculated, for other purposes, the contents of the shapes he deals with, and will, consequently, know at the outset the value of the denominator of second term. For instance, as often occurs, if it be required to find the position of the c. of g. of a piece of iron or timber of known volume or weight and of prismoidal shape, for the purpose of hoisting it, loading it upon a vehicle, or because it is a member of a structure or machine, the formula

$$\frac{(B-A)D^2}{12V},$$

indicating its distance from mid-section toward larger end, is exceedingly simple. If γ be the specific gravity of the material and W the weight of piece, the expression becomes

$$\frac{(B-A)D^2\gamma}{12W}.$$

Even when neither the weight nor volume are known, the practical calculator will find formula (1) very convenient, because it can so easily be reduced to the simplest possible form for special

cases, by mere cancelation of symbols, before the numerical part of the work be commenced.

The calculator can, in most cases, recognize at sight the shapes to which formula (1) applies. For doubtful cases formula (5) is the criterion, and it is easily used. When the shape satisfies this, it is known that both the prismoidal and the center of gravity formulæ apply.

The best practical application of formula (1) is to the determination of the *mean distance*, which the material of a cutting has been hauled to form an embankment, since here it apparently satisfies the greatest want. *Haulage* is the product of the *quantity of material* and the *average haul* or mean distance which it has been transported. *The unit of haulage is one cubic yard hauled one hundred feet.* On this basis the price for haulage is fixed.

After having calculated one factor of haulage, the *quantity*, it remains for us to find the other, or *average haul*. To do this pass a plane anywhere between cut and fill normal to route of haul. Suppose the magnitude of cutting to be generated by a limited plane, whose variable area is represented by y. Let x denote the variable distance of y from the secant plane. However irregular may be the shape of cutting, we know that y varies as a function of x. Hence ydx, the elementary volume, multiplied by x, which produces the elementary amount of haulage, is integrable.

$$\therefore \int_{d_1}^{d_2} xydx,$$

where d_1, d_2, are distances of ends of cutting, is the haulage of cut to plane, and, when x' is the average haul so far as to the plane, the following equation is true.

$$x' = \frac{\int_{d_1}^{d_2} xydx}{\int_{d_1}^{d_2} ydx}. \qquad (18)$$

The first member of this equation is, by definition, the average haul to plane; the second member is, by principles of mechanics, the distance of c. of g. of material from plane, and the equation shows that these are equal.

By similar reasoning it is proven that the average haul from same plane to fill is equal to the distance of c. of g. of fill from plane.

Therefore, *the average haul of a piece of excavation is the distance between the center of gravity of the material as found and its center of gravity as deposited.*

This is as it is stated in works which touch upon the subject. But the practical computation of this theoretical result has been found to be a far more tedious task. It is evident that, first, the centers of gravity of the component solids must be severally ascertained, since the cut or bank is measured as a compound shape. But the application of formula (18) to each of these produces a very intricate expression, involving about double the labor necessary to calculate the true content of the solid by means of the prismoidal formula in crudest shape. After this the several moments must be compounded.

To avoid this some calculators have been in the habit of dividing the excavation into two parts of equal volume by a plane normal to center line, and establishing this as the initial point of the average haul. A plane similarly fixed in the embankment marks the terminal point of same distance. But this plane is always nearer the larger end of shape than the c. of g. is, as may be illustrated upon the cone, triangle or any shape of unequal end dimensions.

For instance, if the first five 100 ft. solids of a railroad cutting have been transported to a bank, and the generatrix commence with an area zero at beginning of cut, and reach, at the end of considered part, an area whose center height is 20 ft., road bed width 20 ft. and side slopes $1\frac{1}{2}$ to 1, the total volume is about 10,000 cubic yards, and, consequently, a difference of 1 ft. in distance makes a difference of one dollar in money. But the difference in the cut is 20 ft., and, if the bank be of same form, 20 ft. is there added to average haul distance. The average haul of other end of same cutting is likely to be also 40 ft. too long. The error of this method, then, makes a total error of eighty dollars for that cutting, which is invariably at the expense of the railroad company. In the time that a calculator would, by this method, compute the haulage of a division of ten miles he would be likely to cost his em-

ployers an amount equal to a year's salary.

To divide the cutting into numerous small parts, and find the sum of their moments, or to determine the c. of g. by a mental estimate merely, are methods either laborious or liable to error.

Because the earthwork solid belongs to the class of shapes bounded laterally by straight line surfaces, formula (1) applies thereto. Let A, B, C, ... K, be the areas of cross-sections, a constant distance apart, 100 feet, or D, in a piece of railroad excavation, whose material is all carried in same direction to form an embankment. Assume an axis, outside of cutting, a distance d_1, equal to 50 feet, beyond A. Then, according to formula (16), the statical moment of first volume, referred to this axis, or the haulage of its material to this axis, is, since $d_1+\frac{1}{2}D=100$,

$$100V_1+\tfrac{1}{12}D^2(B-A), \qquad (19)$$

The haulage of second volume is, evidently,

$$200\,V_2+\tfrac{1}{12}D^2(C-B). \qquad (19a)$$

The haulage of each remaining volume is represented by a similar expression, except that the coefficient of first term is always the product of 100, or D, by the ordinal number of the volume. Thus, for last, or n^{th}, volume, the haulage is

$$100\,n\,V_n+\tfrac{1}{12}D^2(K-J) \qquad (19b)$$

The sum of these expressions is the total haulage to the axis. But the sum of all the second terms is

$$\tfrac{1}{12}D^2(K-A). \qquad (20)$$

Therefore, an exceedingly simple rule for determination of haulage can be constructed.

Before stating this rule let us make a further reduction in the formula. The unit of expressions (19), (20), is a cubic foot hauled a linear foot. To reduce this to the haulage unit, divide by 27D.=2700. Let $Vol_1=\frac{1}{27}V_1$, etc. . . . . $Vol_n=\frac{1}{27}V_n$, that is, let the abbreviations represent the number of cubic yards instead of cubic feet. This, it happens, is the denomination used in dealing with these quantities, and is, therefore, the denomination in which these quantities are presented to us when we commence to calculate the haulage. Now, divide the sum of expressions (19), (20) by 27D, using the abreviations.

$$\left.\begin{matrix}\text{Partial}\\ \text{haulage}\end{matrix}\right\} = Vol_1+2Vol_2+ \ldots . nVol_n + \tfrac{1}{324}(K-A) \qquad (21)$$

Obviously, the axis may be established anywhere. It is merely convenient to place it midway between the 100 ft. stations. It might occupy the position midway between A and B. Then the moment, or haulage thereto, would be expressed by formula (21), with each coefficient of all but final term [called the *correction term*] decreased by unity. So the first term, Vol_1, would vanish.

Since this is so, the embankment can be referred to the same axis, wherever that may be with respect to the bank. In short, all the terms, except the correction term, express the operation, *to multiply the number of cubic yards in each volume by the number of hundred feet that the mid section of that volume is removed from the axis.*

It is well to place the axis between cut and fill, or, if they overlap, as nearly so as possible, in order to avoid negative moments or haulage. For instance, the axis might be established half way between B and C. Then the partial haulage would be

$$-Vol_1+Vol_3+2Vol_4 \ldots +(n-2)Vol_n + \frac{100}{9\times6\times6}(K-A). \qquad (22)$$

This is less than (21), but the difference in defect, 2 *Vol*, where *Vol* is total volume in cubic yards of material removed, is exactly balanced by the same difference in excess, which is created when the haulage from axis to fill is considered.

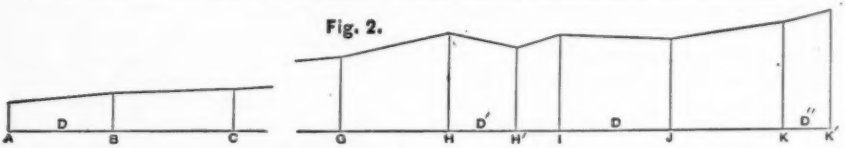
If the irregularity of the ground surface make it requisite to sub-divide one or more of the 100 ft. volumes, as the volume between H and I in Fig. 2, where, also, a plus or intermediate station, K′, instead of a full station, terminates the portion of cutting carried one way; the same rule expressed in italics, second paragraph above, holds, but the correction formula is, for example above,

$$\tfrac{1}{324\,00}[D^2(H+K-A-I)+D'^2(H'-H) +(D-D')^2(I-H')+D''^2(K'-K)], \qquad (23)$$

which is simply a combination of the correction formulæ for the volumes of different lengths.

The haulage from axis to fill is deter-

Fig. 2.



mined in same manner. But the result must be multiplied by

$$\frac{\textit{Excavation}\ \text{Vol.}}{\textit{Embankment}\ \text{Vol.}}$$

for the reason that the material does not occupy same space in embankment as in cut. Ordinary earths become compressed to various degrees. Solid rock fills more space in the bank. The result of division by *Embankment* Vol. is the number of hundred feet from axis to c. of g. of embankment, or the true average haul between. This should be multiplied by *Excavation* Vol., because the amount of material should be measured in the excavation. *The sum of the haulage from cut to axis and from axis to the fill is the total haulage from cut to fill.*

The foregoing may be condensed into the following systematic

RULE.

To find the haulage of material from a piece of railroad excavation to the embankment built therewith, in terms of the haulage unit, 1 cu. yd. hauled 100 ft.

Consider a plane to be passed midway between two consecutive full stations, as nearly as possible between cut and bank.

Multiply the number of cubic yards in each volume of full [100 *ft.*] *or minor* [*less than* 100 *ft.*] *length, in cutting by the number of hundred feet its mid-section is removed from the plane. If any such mid-section be on the side of plane toward the fill, its product must be taken as negative. The sum of these products is approximately the haulage from the cut to the plane.*

To correct this add the expression

$$\frac{\text{D}}{9\times6\times6}\,(\text{K}-\text{A}),$$

once for every series of consecutive volumes of equal length, in the cutting, D *being the length of each such volume,* K, *the area of end cross-section of the series, further from the plane, and* A, *the area of end cross-section nearer the plane.*

Determine in exactly same manner the haulage from the assumed plane to the fill; but divide result by number of cubic yards in bank, and multiply quotient by number of cubic yards in cutting. The sum of these is the total haulage required.

This rule, although wonderfully simple, in view of what might be expected from so irregular a solid as a railroad cutting, is absolutely correct. It only remains to multiply its result by the price paid per unit of haulage to find the price to be paid for the work.

The position of c. of g. of cut may be determined in same manner described above for the bank.

It frequently occurs that portions of excessive cuttings are transported to spoil-banks near at hand. Often the entire top is taken off by scrapers. The accurate final estimate does not distinguish between these portions. In such a case the haulage from cutting to fill, as found by rule, is not the true haulage. Let *Vol'* be the amount in cubic yards wasted, as recorded in monthly estimates. Let H′ be the haulage of this amount to waste pile, as determined in monthly estimates. The position of c. of g. of this portion of cutting must be known. It can be found, according to method of last paragraph, at the time when that material is measured, and its position should be recorded. Let the distance of this position from the c. of g. of fill be L′. Now, the error in the haulage, as first calculated, is the result of the operation founded on the supposition that Vol′ was moved to the fill instead of to the waste bank. The correction is, in consequence,

$$\text{H}'-\text{L}'\times\text{Vol}'. \qquad (24)$$

Quite as often it happens that portions of the embankment are built of material from borrow-pits at hand. Let *Vol''* be the number of cubic yards borrowed, as measured in fill, H″ the haulage thereof, and L″ the distance of c. of g. of this portion of fill from the plane. Let the number of cubic yards in entire part of bank to which material from cut has been hauled be *Vol'''*. This is equivalent with expression, *Embankment* Vol, used above. Then (*Vol'''* − *Vol''*) is the portion of bank brought from cut; and its c. of g.,—not

the c. of g. of Vol''', as calculated in rule, —is the terminus of average haul distance from cutting. If L''' be the distance of c. of g. of fill from plane as determined in rule, and L^{iv}, the distance therefrom of c. of g. of $(Vol'''-Vol'')$, then

$$Z^{iv}=\frac{L'''Vol''-L''Vol''}{Vol'''-Vol''}. \qquad (25)$$

The correction to be made in mean haul distance from cutting is $(L^{iv}-L''')$; and the correction to be made in haulage is, when a portion Vol' of cut, has been wasted,

$$(L^{iv}-L''')\ (Vol-Vol') \qquad (26)$$

$$=\frac{(L'''-L')Vol''}{Vol'''-Vol''}(Vol-Vol') \qquad (27)$$

When nothing has been wasted, Vol' in (26), (27) is zero.

The haulage H'' from borrow-pit is usually kept separate.

Formulæ (26), (27) are correct on the supposition that the correction (24) has already been made.

It appears from the foregoing that in cases where parts of cuttings are wasted, or parts of embankments are borrowed, not only should the quantities and haulage of such parts be estimated from monthly measurements, as is always done, but also the centers of gravity of these parts should be established and recorded at such times, for use in determination of total haulage finally.

The same methods apply, of course, to the borrow-pits and waste-banks. The method can be readily modified to suit all practical cases.

For another method, also depending upon formula (1), of determining haulage, when the cuts and banks have been calculated entire, that is, when the contents of single volumes are unknown; also, for graphical methods of solving the problems just presented, and for details concerning the terminal solids of the banks and cuts, the reader is referred to the chapter on *Average Haul in Formulæ for Railroad Earthwork, Quantities and Average Haul*, since these depend partially upon formulæ foreign to the nature of this article.

The plan of basing contracts upon excavation and haulage prices, seems to be preferred to that which considers excavation and embankment prices. The advantage of the former is that any difference in labor, occasioned by change in amount of haulage, so often made necessary during progress of work, is directly provided for in the original agreement.

The paragraphs containing expressions (19), (20), show that the statical moment of a series of consecutive, equally long shapes, each of which is represented by some of the forms of a quadratic function, we may find by assuming the c. of g. of each shape to be half way between its ends, then compounding the several moments, and finally correcting by the expression

$$\tfrac{1}{12}D^2(K-A). \qquad (20)$$

To determine the c. of g. of series, divide foregoing result by V. Therefore, find the c. of g. of series on supposition that the c. of g. of each shape is in its mid-section, and correct by adding to the distance, of the point thus found, from A the following:

$$\frac{D^2(K-A)}{12V}. \qquad (28)$$

Thus, formula (1) reduces the problem to this simple one: to find the resultant of a system of parallel forces in one plane, whose intensities and positions are given, the position of this to be corrected by expression (28). The singular advantage of formula (1) is, that its second or correction-term, (28), remains as simple for any number of shapes in the series as for one.

It is evident, in consequence, that the error of the assumption that the c. of g. of each shape is in its mid-section, is comparatively less as the series is longer; also, that no error whatever results from this assumption, when the end areas are equal.

For instance, to find c. of g. of a spherical sector whose component cone and segment have equal altitude, it may be assumed that the c. of g. of each is half way between its bases.

To find c. of g. of a series of trapezoids such as represented in Fig. 2, assume as before the c. of g. of each to be midway its length, and correct the resulting position of c. of g. of series by

$$\frac{D^2(H+K-A-I)+D'^2(H'-H)+(D-D')^2(I-H')+D''^2(K'-K).}{12\,V} \qquad (29)$$

The series of trapezoids may have such arrangement that some are negative, as

illustrated in Fig. 2, p. 291, April No. of this Magazine. When the last ordinate is coincident with first, the algebraic sum of trapezoids is the area of a polygon, included by the top lines of the trapezoids, and the c. of g. or statical moment of this polygon can be found by application of formula (1) or (16).

Suppose the polygon to be ABCDEA. Let a, b, etc., be the ordinates of the vertices; and let a', b', etc., be the corresponding abscissæ of the same. Since we may at first assume the c. of g. of each trapezoid to be midway its length, the several moments of these figures, when the lever arm lies in the direction of abscissæ, are,
for first,

$$\tfrac{1}{2}(a+b)(b'-a')\tfrac{1}{2}'(b'+a') = \tfrac{1}{4}(a+b)(b'^2-a'^2), \quad (30)$$

for second,

$$\tfrac{1}{4}(b+c)(c'^2-b'^2), \quad (30a)$$

. . . . . .

for last,

$$\tfrac{1}{4}(e+a)(a'^2-e'^2). \quad (30b)$$

The correction term, for first trapezoid, is

$$\tfrac{1}{12}(b'-a')^2(b-a); \quad (31)$$

. . . . . .

for last,

$$\tfrac{1}{12}(a'-e')^2(a-e). \quad (31a)$$

The sum of expressions (30), (31), is the statical moment of polygon, when the lever arm is parallel to abscissæ. This may be arranged as follows:

$$\tfrac{1}{6}[a(b'-e')(e'+a'+b')+b(c'-a')(a'+b'+c')\ldots+e(a'-d')(d'+e'+a')]. \quad (32)$$

To obtain abscissa of c. of g., divide (32) by area of polygon, as expressed by rule B, p. 293, April No. of this Magazine. Accordingly, distance c. of g. of polygon is

$$\frac{\tfrac{1}{6}[a(b'-e')(e'+a'+b')+b(c'-a')(a'+b'+c')\ldots+e(a'-d')(d'+e'+a')]}{\tfrac{1}{2}[a(b'-e')+b(c'-a')\ldots+e(a'-d')]} \quad (33)$$

Each term of the statical moment is the product of three factors, two of which are the factors of a term of the area. These two factors need be used once only. This makes the determination easy.

Always, when choice may be had, place the origin at one vertex, as at A. Then $a=o$, $a'=o$. In consequence, one term in the numerator, and one in the denominator, of fraction, vanish, and some of the remaining terms become reduced.

Each term of the statical moment is the continued product of the ordinate of each vertex, the difference between the abscissæ of the two adjacent vertices, (the subtraction being made always in same direction around polygon,) and the sum of the abscissæ of the vertex itself and the two adjacent vertices.

To construct the formula for the ordinate of c. of g., simply change, in formula (33), *a, b, etc., to a', b', etc., and a', b', etc., to a, b, etc.*

Professor Weisbach in article 112 of his *Theoretical Mechanics*, Eckley Coxe's translation, demonstrates a method of determining c. of g. of polygons. The one above presented is, however, decidedly shorter when one co-ordinate only of the c. of g. is required. When both co-ordinates are sought, there is little preference between them, this being in favor of Weisbach's method.

The first two columns of the following example are quoted from the article in *Theoretical Mechanics* above referred to. The problem is solved by the method of this paper.

a'	a	Double Area.	Sextuple Stat. Moment.
24	11	× 11=121	× 49= 5929
7	21	× 40=840	× 15=12600
-16	15	× 19=285	× -21=-5985
-12	- 9	× -34=306	× -10=-3060
18	-12	× -36=432	× 30=12960
		1984	22444

$$x_1=\tfrac{1}{3}\cdot\tfrac{22444}{1984}=3.771.$$

To find y_1, substitute a for a', and a' for a, etc.; then proceed as in above example.

Applied to a triangle ABC, when the origin is placed at vertex A, formula (33) becomes

$$\frac{\tfrac{1}{6}(lc'-cb')\ (b'+c')}{\tfrac{1}{2}(bc'-cb')} = \frac{\text{Stat. Mom.}}{\text{Area.}} = \tfrac{1}{3}(b'+c')=x_1, \quad \text{Likewise,} \quad \tfrac{1}{3}(b+c)=y_1, \quad (34)$$

The abscissa and ordinate of c. of g.

of the retaining wall, Fig. 3, the origin being at the left end of lower base, are:

$$x_1=\frac{\frac{1}{6}[hd'(d+d')+h'(w'-d)(d+d'+w')]}{\frac{1}{2}[hd'+h'(w'-d)]} \quad (35)$$

$$y_1=\frac{\frac{1}{6}[-dh'(h+h')+d'h(h+h')+w'h'^2)]}{\frac{1}{2}[-dh'+d'h+w'h']} \quad (36)$$

The numerator of (36) can be reduced.

For Fig. 4,

$$x_1=\frac{\frac{h}{6}[d'(d+d')+(w'-d)(d+d'+w')]}{\frac{h}{2}(w+w')} \quad (37)$$

According to formula (1),

$$y_1=\tfrac{1}{2}h+\frac{(w-w')h^2}{12V}; \quad (38)$$

or, from (36), when $h=h'$,

$$y_1=\frac{2w+w'}{w+w'}\,\frac{h}{3} \quad (38a)$$

Fig. 3.

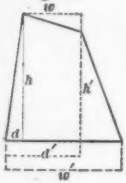


Fig. 4.

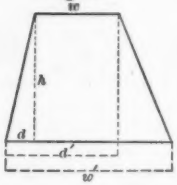
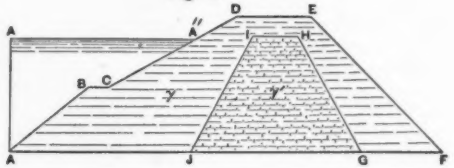


Fig. 5.



To determine the abscissa of the c. of g. of the embankment, Fig. 5, which has a core of different density from that of the covering material, we may find the abscissa of polygon ABCDEFGHIJA, and then that of polygon JIHGJ, and compound the moments, having regard to the unequal densities. But the problem can be solved more simply in one operation. Suppose γ to be the heaviness of covering material, and γ' the heaviness of core material. Now consider at once the polygon ABCDEFGHI JIHGA, regarding the ordinates of I and H, the second time used, as $\frac{\gamma'}{\gamma}\,i$, $\frac{\gamma'}{\gamma}\,h$.

To obtain the ordinate of c. of g. of whole dam, consider the same complete polygon, but regard the abscissæ, when used second time, of all the vertices belonging to the core [J, I, H, G], as if they were longer in ratio of γ' to γ.

If it be desired to ascertain the common c. of g. of the water resting on slope, and the dam itself, consider the polygon, AA'A''CBA, BCDEFGHIJ, JI HGA, and regard, when determining the abscissa, the ordinates between first and second commas as multiplied by γ, and those after second comma, as multiplied by γ'. When determining the ordinate of c. of g., multiply the corresponding abscissæ by same quantities.

For the determination of the statical moment, or the c. of g., of any series of trapezoids, formula (32) or (33) is preferable to formula (16) or (1), when the lengths are all different. This is very apt to be the case in that particular arrangement of trapezoids which forms the polygon. (32), (33) are, therefore, eminently suitable to this shape. Whenever a majority of the trapezoids have equal lengths, as in Fig. 2, formula (1) is to be preferred.

As it usually happens that the areas and volumes of practical shapes are calculated for other purposes in advance of the determination of centers of gravity, formula (33) is generally in simplest form. When the area of polygon is neither known nor required, expression (33) can be always reduced by cancelation of common factor $\frac{1}{2}$, and frequently can be much further simplified, as in case of triangle, and to form expressions (37), (38*a*).

The denominator of (33), when that formula expresses the abscissa, is equal with in value, though different in form from, the denominator of (33) when it expresses the ordinate of c. of g. Therefore, the denominator need be calculated once only. Thus the denominators of (35), (36) are the same in value, each representing the area of the polygon.

THE ÆSTHETICS OF ENGINEERING.

By RICHARD GERNER, M. E.

Written for VAN NOSTRAND'S MAGAZINE.

By way of repeating a frequently quoted remark, with a view of impressing its significance upon the reader's mind, be it stated that there is no profession which is so interwoven, not only with other professions, but also with the more ideal and oftentimes fanciful walks of the human mind, as that of the mechanical engineer. We see the engineer's handiwork in that of the architect, the builder, and, in brief, the representative of almost every industry, trade and profession. This is only natural, for wherever ingenuity is called into play, wherever primitive nature is to be reconstructed for the practical comforts of mankind, wherever necessity or ambition drives the human mind into the realms of mechanical genius, the engineer is at his station,

> "Placed on this isthmus of a middle state,
> A being darkly wise, and rudely great."

There was a time when his services were required solely in the solution of practical life problems—as simply to span the river with a bridge, and myriads of similar questions.

> "Great nature spoke; observant man obeyed,"

and civilization grew out of primitiveness.

But now that we are established, now that our wants are supplied, now that we are at leisure to turn from the merely problematic to the consideration of work of improvement, let us lend an ear to the dictates of the mind, the heart and the soul; and while continuing in our work to minister to the bodily wants of mankind, see if we cannot apply our work to the aggrandizement of the beautiful and the sublime, or, vice versa, the flights of art, imagination and poetry, to the elevation of our work.

You would not have, rising on a foaming, craggy promontory, a huge, rude tripod, supporting a torch to warn the storm-tossed mariner off the breakers, or a few planks thrown carelessly across a stream to afford a transit, or a clumsy pipe sticking out from a basin to represent the fountain of a garden, or a bell tolling from an uncouth gallows to call the pious to their devotions.

Each serves the purpose for which it was intended, it is true; mariners would not be safer, transit less difficult, jets of water less brilliant or bell peals less inspiring, were it otherwise. But it is an outrage upon our sense of the beautiful.

What you want is to behold a Bedloe's Island Gift of France rising from every surging shore, a Girard Avenue Bridge over every stream, a Loreli or Neptune in bronze splashing from every fountain basin, and a Trinity Church steeple everywhere to proclaim the hour of devotion.

How uncivilized they would feel in Philadelphia, had they to miss their Fairmount Water Works, and their water to be taken from the Schuylkill and the Delaware, borne in barrels on mules through the city and sold for a penny a gallon to the inhabitants, as is now practised in the far Orient, and the land of the Southern Cross; how unprogressive we should feel, in New York, had we to want our gas works, and to light up our proud metropolis with small oil lamps, secured on wooden posts, as now yet practised half the civilized world over.

How odious such comparisons. And yet, those who do not possess these advantages cannot appreciate what it is to want them; people never miss what they never had; they feel comfortable in their misery.

> "See the blind beggar dance, the cripple sing,
> The sot a hero, lunatic a king;
> The starving chemist in his golden views
> Supremely blest, the poet in his muse."

And again, do you doubt that our posterity will make such comparisons between their state of civilization and ours? Will the Captain Nemos and the Professor Wises of those days not smile in derision upon the diving bell and the balloon ascensions of the nineteenth century? Will the engineers and builders of the great unknown future of mankind not jest over the primitiveness of this age, in contemplating the colossi of New York, San Francisco, Yokohama, London and where not? The Island statuary Gift of Greece to France, of Italy to Germany, of Spain to England, etc.? The Miltonian tunnels through the An-

des, the Himalayas, the Rockys and the Alps, resembling gates to the Infernal Regions, on whose hinges you might fancy hearing

"— grate
Harsh thunder, that the lowest bottom shook
Of Erebus"

for electric locomotives to flash through? the stupendous structures over the rivers, as the Eagle spreading its bronze wings over the Hudson; the Crescent stretching its arc over the Dardanelles, the Lion leaping the Thames, or the Tea Leaf Bower gracefully spanning the Yangste-Kiang? Shall we not suffer ridicule and humiliation?

Shall that ever be with us? No! Then teach your engineers the legends of all nations and climes, mount where fancy guides, study the dreams of our poets, observe the thoughts of our artists, follow where history leads, and lift your gaze into the clouds, so that Alad din may be seen standing on some rocky isle, illuminating with his wonderful lamp the gloom of the dangerous waters, and Lohengrin ascending the Hudson, bearing a pleasure party in his shell, so that dragons may fly over the rails, on festive occasions, and ice bears spit the sparkling soda; so that our cities may be graced with Vulcans supplying us with gas, and Rebeccas with water, and so that the splendors of the empires of romance and reality may be symbolized to beautify our works of to-day.

Why should things be done clumsily when they might just as well be done handsomely? A nation in whom the sense of the beautiful is deadened, is little more than a colony of barbarians.

But it must be remembered that there is a marked line between going to work in a reasonable manner and going into extremes. There are instances where the old epigram, "beauty unadorned, adorns the most," finds a remarkably good application.

Take the steam engine of to-day. Observe as it exists in its height of perfection, how graceful its proportions, how beautiful its curves, and how tasty its form. How proudly the engineer gazes upon that child of his brain. Would he feel the better, do you suppose, were the fantastic dreamer to seize upon its symmetry and simple charm, and transform the piston-rod into the sword of the Duke of Glo'ster, and the cylinder into the aged breast of Henry VI? Not while such designers as Mr. Babcock or Mr. Corliss are living to give life to such engines as we have seen displayed at our Centennial Exhibition and elsewhere. What exquisite regularity and grace of motion; there is almost poetry in it. There is as much food for the mind of the dreamer in the engine-room of a steamer as in a romantic wilderness, where, in the former case, he is tranquilized and wrapped up into a delicious brown study in the observation of the regular, alternate, up-and-down stroke of the steel bars in their journals, and in the latter case, perchance, in the dripping of successive crystal drops from out of a rock into a tiny pool.

Who is there that can stand within a leap of a railroad track and not feel stirred into the innermost confines of his feelings, with the sublime grandeur that takes momentary possession of his entire self when the locomotive dashes by?

Who is there so hardened to fancy that he cannot penetrate into a gloomy tunnel and not imagine himself on his way to fantastic abodes: of the throne of the monarch of the dwarfish miners, of the old Teutonic legends, of the chamber of the Queen of night, or of the realms of Satan himself?

But it were purposeless to cite further instances; go where you will, into any department of engineering, you will encounter appeals to your admiration, your wonder, your emotions, and, perhaps, your passions.

Whence we observe that, of all things, engineering is less incompatable with the beautiful and the sublime than many believe, and is more indispensable to the furtherance of the beautiful, the sublime and the poetical than many know.

Of this latter we have a striking exemplification near at hand. All the splendor, the beauty, the wonder and the grandeur of the spectacle of *Baba*, lately on the boards of the New York stage, where would it be but for the engineer? People go there and praise the author for the charming fairy tale he has told us, and the artist for the magnificent illustratory scenes he has painted us, without taking into consideration that, but for the genius of Mr. Sherwood, the designing engineer, the literature and

the art of the play would all be in vain, towards approximating to the results attained.

It is in this manner that we see engineering play its part in every department of human activity; even in the activity of the imagination, and we see the utility of the engineer being versed in art, romance, etc., no less than the artist or the scholar. He cannot be too thoroughly acquainted with any department of human learning; his profession is at once an art, a science, and an industry.

It should be remembered that it is not so much the detail beauty of a structure which wins a reputation for the author, but the boldness of its conception; and and boldness in this regard is an attribute of that mind which has roamed unrestricted through all the confines of knowledge.

Decoration in detail is very good in some instances, and has won a name for many a designer, but it will be observed that the greatest triumph is reserved for him who at once conceives an ornament in the construction of the whole.

As, for instance, to construct a bridge in a mountainous or rocky region after the model of a bridge of the middle ages, one of those immense, gaunt, grim structures, which look so well on canvas, and which would be so proper on some rivers adapted by nature to be associated, in its surroundings, with such a bridge.

Beauty in structures to-day depends somewhat more upon style and taste than on fanciful conception. Everything is aimed at simplicity; designs lose the intricacies of those of the sixteenth and seventeenth centuries; surfaces are smooth to-day in contrast with the heavy decoration of those days; in many cases, the simplicity has become almost to severe.

This is but an era of style, as there have been so many eras, and so many as there are yet to come, and the simplicity of to-day, while it may find resurrection in some future age, may be replaced to-morrow by that ponderous ornament which *it* replaced yesterday.

The engineering fashions of to-day seem very much in accordance with the general intellectual levity of the age; the outward *tout ensemble* of our metropolis speaks of easy elegance rather than of solid magnificence. As an example may be cited the Girard avenue bridge in Philadelphia, and also the latter-day park and illumination improvements in New York. They are entirely in harmony with the character of gay flashy capitals. London, Paris, Vienna, Berlin and St. Petersburg look to-day as they did a century or two ago, as much as the gas fixtures of those days look like the gas fixtures of to-day.

The Fairmount Bridge at Philadelphia is an attempt at a contrast, and was built on an entirely different conception. Unfortunately, it is partly a failure. Mr. A. P. Boller, in his work on "Iron Highway Bridges," calls it a "huge architectural fraud," and with justice, for "instead," Mr. Boller says, "of letting the enormous trusses stand in all their grandeur, depending wholly upon judicious painting and the design of the cornices and railing for their æsthetic effect, thousands of dollars have been spent in actually covering up the trusses, to a great extent, with sheet-iron, forming an arcade, as it were, of great massiveness, by arching between the posts of the trusses, the arches springing from large Roman sheet-iron capitals, about halfway down the posts! The result is that, at a little distance, the spectator beholds an arcade, without any visible means of support, for 340 feet. To be thoroughly consistent, the architect of this constructed decoration should have at least sanded his sheet-iron when painted, and marked in strong lines the joints that masonry of similar forms suggests."

The bridge is built in violation of the Ruskinian axiom: "Decorate the construction, but do not construct decoration." "Such a principle conscientiously kept in view," continues Mr. Boller, "can only result in good work. Its violation, in a majority of cases, results in a demoralization of the taste of the community. Public works, in a sense, play a part in the education of a people, and their authors and builders consequently have a responsibility in addition to the mere utilitarian idea of endurance and safety."

The axiom just enumerated can only violated under peculiar circumstances, and then only by men who have their wits about them. An error is fatal, as

in the case of the Fairmount Bridge. In all other respects it "is a monument to its designer and an honor to American engineering."

Ruskin, be it stated, in his axiom, did not mean to say that a thing was not to be constructed because it was an *ornament*, such as a statuary light-house. The terms *ornament* and *decoration* are not convertible; *decoration* is that which serves to beautify, adorn or embellish something else, such as a structure; an *ornament* is a *decoration* in itself, or, perhaps, a *decoration* of the general surroundings; thus, a frescoed ceiling is a *decoration*, while a fountain is an *ornament;* an *ornament* may be *decorated*, but never a *decoration ornamented.* Ruskin referred strictly to *decorations.* Webster does not make this distinction, but I think I shall be sustained in making it here.

Another departure from the style of the age is rustic work, and the specimens in the Central Park of New York furnish excellent examples of the beauty and practicability of such work in bridges, steps, cottages and railings.

Style, in construction, depends in a great measure upon the materials at the command of the engineer. The somberness of the style of the middle ages was due as much to the stone then at their command as to anything else. And so it was in ancient history. Rome was built essentially of marble and granite, and hence, in great massiveness. We do not depend upon stone only to-day. Our progress in chemical and physical science has given us the metals and other material, and the result is necessarily lightness in construction. They have aided us in conceiving of bolder engineering projects, but they have not by any means reached the maximum of their adaptability. Only when they are utilized in the repetition of the engineering wonders of the ancients, and when our cities partake of the magnificence and grandeur of the capitals of Nero, Sardanapalus and others, shall this have been attained. And with our facilities, we shall be able to remodel the appearance of our surroundings in a manner that will outvie the fondest conceptions of the boldest and most extravagant dreamer.

THE BLUE PROCESS OF COPYING TRACINGS.—In a short paper read before the Institute of Mining engineers Mr. P. Barnes gave the following summary of the manipulations required in this process: 1. Provide a flat board as large as the tracing which is to be copied. 2. Lay on this board two or three thicknesses of common blanket, or its equivalent, to give a slightly yielding backing for the paper. 3. Lay on the blanket the prepared paper with the sensitive side uppermost. 4. Lay on this paper the tracing, smoothing it out as perfectly as possible so as to insure a perfect contact with the paper. 5. Lay on the tracing a plate of clear glass, which should be heavy enough to press the tracing close down upon the paper. Ordinary plate-glass of $\frac{3}{8}$ in. thickness is quite sufficient. 6. Expose the whole to a clear sunlight, by pushing it out on a shelf from an ordinary window, or in any other convenient way, for six to ten minutes. If a clear skylight only can be had, the exposure must be continued for thirty or forty-five minutes, and under a cloudy sky sixty to ninety minutes may be needed. 7. Remove the prepared paper and drench it freely for one or two minutes in clean water, and hang it up by one corner to dry. Any good hard paper may be employed (from even a leaf from a press copy-book up to Bristol board) which will bear the necessary wetting. For the sensitizing solution take $1\frac{7}{8}$ oz. citrate of iron and ammonia and 8 oz. clean water; and also, $1\frac{1}{4}$ oz. red prussiate of potash and 8 oz. clean water; dissolve these separately and mix them, keeping the solution in a yellow glass bottle, or carefully protected from the light. The paper may be very conveniently coated with a sponge of four inches diameter, with one flat side. The paper may be gone over once with the sponge quite moist with the solution, and a second time with the sponge squeezed very dry. The sheet should then be laid away to dry in a dark place, as in a drawer, and must be shielded from the light until it is to be used. When dry the paper is of a full yellow or bronze color; after the exposure to the light the surface becomes a darker bronze, and the lines of the tracing appear as still darker on the surface. Upon washing the paper the characteristic blue tint appears, with the lines of the tracing in vivid contrast.

THE Zabiaka, the fourth and last of the Russian cruisers, built at Philadelphia, is reported to have averaged over fifteen knots an hour on her trial trip. The Zabiaka is 220ft. long, has 30ft. beam, moulded, 17ft. 6in. depth of hold, and has a mean draught of 11ft. 9in. She is bark rigged and carries a great deal of canvas; is about the same length as the Alabama, and is said to look like that famous privateer externally. The battery consists of six rifled breech-loading guns of Russian manufacture, sent from St. Petersburg, and made of cast steel. Four are of 4in. bore and are broadsides, the other two are of the 6in. bore and are pivot guns. All are to be mounted on the spar deck, and the pivot guns will train in any direction. The interior arrangements of the Zabiaka are—according to the American *Army and Navy Journal*—models of fine workmanship and convenience.

ENGINEERING GEOLOGY.

By W. H. PENNING, F.G.S.

From "The Engineer."

I.

In the execution of engineering works, however scientific in design and clever in workmanship, failure has frequently usurped the place of success, because due attention has not been paid to geological phenomena. The same may be said of building operations; whilst it is notorious that vast sums of money have been thrown away in mining speculations which would at once have been characterized as hopeless by any one possessing the slightest acquaintance with geology. Although a knowledge of this science is undoubtedly a great acquisition, which affords both pleasure and profit to the possessor, it is not possible, nor even desirable, for all professional men to become proficient geologists. Those for whom these papers are more especially intended have too many claims upon their time and attention to bestow either upon a study of abstract principles, laws, and theories, which do not relate to their own particular science, art, or occupation; but they may with advantage avail themselves of the labors of others, when the results of those labors bear directly and in a very important degree upon the stability or success of the works whereon they are engaged.

The engineer certainly should make himself acquainted with the geology of a district through which a line of railway is to be constructed from his designs and under his superintendence. He should ascertain the nature of the various rocks that will be met with, not only at and near the surface of the ground, but for a considerable distance below; their relation to each other, and the consequent influence they will exert over the works in contemplation. "Trial-holes" are generally dug for this purpose, but these are simply pits excavated to a depth of a few feet, and afford information which, although valuable in itself, extends only to the superficial deposits. Borings are sometimes made, but are, in most cases, too costly; and however numerous these, or trial-holes may be, both fall far short of what can be achieved in the same direction through the methods employed by the field-geologist. By these he determines not only the kind of rocks occurring at or near the surface, but also their position in regard to each other; and is enabled to indicate with reasonable accuracy what strata will be met with, and in what successsion, to a depth, it may be, of several hundred feet; and these results of his labors include not merely a knowledge of what beds would be pierced in sinking a well, or in excavating trenches for foundations, such as would be afforded equally by trial-holes or borings of sufficient depth; but they embrace also the important points of the "lie" of the beds, the order of their superposition, their outcrop, dip, and consequent water-bearing properties; by all of which the stability and durability of engineering works are greatly affected. One cannot fail to perceive how differently placed or constructed would have been many of the most important works, such as fortifications, railway-cuttings, embankments, tunnels, and even sewers, had those who designed them been acquainted with the principles, methods, and results of field-geology; or how much capital might have been usefully instead of fruitlessly expended, or how many catastrophes would have been averted. Mention has already been made of costly sinkings for minerals, where they could not possibly have been found; large sums of money have also been wasted in equally fruitless searches for water. Yet water-supply is as amenable to known laws as any other phenomenon of Nature, and within certain limits it may be determined without experiment. Although the divining rod has not even yet ceased to be a power amongst foolish people in certain localities, its days are surely numbered; men must, sooner or later, come to see that springs are merely water finding its own level, and that for water to issue forth at one part of the earth's surface, it must

have been absorbed at another. When the conditions affecting its absorption by and passage through the strata of a district are known or can be discovered, the existence of springs, their depth from the surface, and the height to which they will ascend, can be approximately, if not with extreme accuracy, determined. In his "Rudimentary Geology," Major-General Portlock has truly and eloquently said, "Geology is now a true science, being founded on facts and reduced to the dominion of definite laws, and in consequence has become a sure guide to the practical man. The miner finds in it a torch to guide him, in his subterranean passage, to the stratum where he may expect to find coal or iron, or to the recovery of the mineral vein which he has suddenly lost; the engineer is guided by it in tracing out his roads or canals, as it tells him at once the firmest stratum for supporting the one, and the easiest to cut through for the other, and makes him acquainted with the qualities of the materials he should use in his constructions, and the localities where he should seek them; the geographer finds his inquiries facilitated by learning from geology the influence of the mineral masses on the form and magnitude of the mountains and valleys, and on the course of rivers; the agriculturist is taught the influence of the mineral strata on vegetable and animal life, and the statesman discovers in the effects of that influence a force which stimulates or retards population; the soldier also may find in geology a most valuable guide in tracing his lines both of attack and defense; and it is thus that a science rich in the highest objects of philosophical research is at the same time capable of the widest and most practical application."

In the following papers rules and methods relating to stratigraphical geology only are given; as the geological conditions which affect engineering and similar works are the extent of the various strata, their lithological character, and order of succession. It matters not what may have been the forms of Life during the ages when the strata were deposited, what their relations to those older or more recent, or what the order of their appearance in time; although the evidence upon these questions is as strong and as interesting as any upon which is based the science of geology. The rocks will be treated merely as stones, clays, and sands of varying quality; some possessing commercial value and great utility; others having qualities to be guarded against in all mechanical operations; some only exhibiting water-bearing properties; but all worthy of study, independently of the old-world histories which they contain. The names of places are given only in particular instances, such as those of mines, important quarries, notable sections, and so on, it having been considered advisable not otherwise to refer to localities in the description of the rocks. These are mentioned generally, and under specific denominations geological maps indicating much more readily the formation at any particular spot than a lengthy reference to the many places which must otherwise have been mentioned as situated on an extended outcrop. The main object of these papers is to enable the observer to discover and for all practical purposes to trace out for himself the nature and extent of the rocks with which he is concerned. The "drifts" are usually omitted from the maps; these are a series of superficial deposits not shown on any of the older geological charts, and noticed on only a few of the more recent official publications. They consist chiefly of clays and gravels of peculiar character, which are found here and there upon the older rocks, on hills and in valleys, with no very definite mode of occurrence. A section is devoted to a brief description of these deposits, with the methods of tracing and mapping them, as they must be treated from a practical point of view, in the same way as the older and more important formations.

Geological Strata: Their Nature, Relation, and Bearing upon Practical Works.—The earth's crust consists of a great number of alternating rocky layers, various in kind, thickness, and extent, but always in regular, if not in constant, sequence. The uppermost have been formed in a great measure from the waste of those beneath, in the same way as the material now being deposited on the bottom of the sea has been derived from the denudation of the existing dry land. These layers are but rarely hori-

zontal, and they bear evidence of having been subjected to an upheaving force which has acted at various times, unequally and with different degrees of intensity, beneath every portion of the earth's surface. There have been corresponding, and on the whole nearly equal, movements of depression, and all areas have frequently been dry land, again to be covered by the waters of the ocean. It is owing to this inequality in the upheaval of the beds, and to their consequent partial destruction by the sea, that the lower and older strata are now exposed at the surface of the ground, and that we are enabled to classify the rocks and to decipher their ancient history. The formations, of which the denuded edges are thus bared and thrown open to our inspection, are indicated by different tints upon geological maps. If it be borne in mind that each of the areas thus distinguished represents, as a general rule, the edge and not the surface of a formation, the proper understanding of a geological map is much facilitated. It is evident that were these variously-colored portions each indicative of an original surface, the rocks so depicted would generally be the newest, as overlying those which are hidden beneath. But their edges only being exposed and portrayed on the map, the planes of bedding must be either in a vertical position, or inclined from the surface in some direction, and the rocks, as a matter of course, must then pass in under some of those that are contiguous. Any geological map shows that, in this country, by far the larger proportion of these edges, or lines of outcrop, follow a nearly north and south direction, therefore the beds must dip, if at all, either to the east or to the west. The general dip of the formations in these islands is towards the south-east; consequently those on the north-west are the oldest, and the lowest in the geological scale; those on the south-east are the highest in the scale, therefore the most modern. All the beds of which the various formations are composed are termed "rocks," whether they be hard or soft, of aqueous or igneous origin. The following remarks upon them have been as far as practicable classified under three headings—(*a*) The nature of the rocks; (*b*) Their relation to each other; (*c*) The bearing of the nature and relation of the rocks upon practical operations.

(*a*) *The Nature of the Rocks.*—The rocks of which the known crust of the earth has been built up, in successive layers, are of infinite variety as regards texture, color, hardness, and other peculiarities. All are made up of minerals either in a crystalline or fragmentary form, or of mineral matter in a state of comminution. Some rocks contain metals, either free, or, as ores, in combination with oxygen, and but comparatively few are without metallic coloration. All rocks may be divided into two great classes:—(1) The igneous or unstratified, which were formed below the surface, and were by volcanic or similar force erupted through or intruded into the pre-existing formations. (2) The aqueous, or stratified, which were deposited from water as sediment, or in some cases as chemical precipitate. There are rocks which are otherwise formed, and some which have been altered from their original condition by heat or pressure, or by both agencies combined; but in a work of this nature the scientific differences may be disregarded. (1) The igneous rocks are the granites, dolomites, diorites, felspars, basalts, trachytes, and tuffas. (2) The aqueous are sandstones, clays, limestones, flints, and gravels. The altered, or metamorphic rocks may have been either igneous or aqueous, but were principally the latter, and are now found as gneiss, quartzites, clay slates, schists, and altered limestones. The class to which rock belongs is practically important, on account of the difference in the normal mode of its occurrence. The stratified rocks lie evenly, the one upon the other, and preserve a regular but sometimes interrupted sequence; the unstratified follow no definite lines, but suddenly break through older rocks and disappear in an equally abrupt manner. In both classes the rocks of every kind present many varieties and gradations towards each other, but on the whole they possess broad characteristics by which they may be fairly determined. It may be noted that, generally, but not without exceptions, the older stratified and the altered rocks are more crystalline and compact than those of more recent date. Those that were by

an old classification designated primary, consist of slaty and crystalline strata, such as gneiss, and mica-schist, marble, and clay slate; transition, of slaty and siliceous sandstones and calcareous shales; secondary, of chalk, limestones, red sandstones, marls, and clays; tertiary, of sands and clays; recent, of sands, gravels, silt, peat, and alluvium. The loose and friable beds are the most recent, overlying others more consolidated of secondary age, which in turn rest upon the older and more crystalline strata. All were once in the same unsolidified condition, but some have become hardened during the ages which have elapsed since their accumulation.

(*b*) *The Relation of the Rocks to Each Other.*—For mechanical purposes, the relation of the rocks of a district is quite as important as their individual nature, but not so readily ascertained, unless the proper methods of procedure be understood. Their relative thickness, of course, rules the extent of the ground occupied by each, but it must be studied, also, in connection with their dip, which exercises a powerful influence on the shape of the country where they come to the surface. Where the bedding of rocks is horizontal, or nearly so, the surface will be much more flat and spreading than where the dip is sharp, which will produce a rugged and rapidly-alternating landscape. This fact is well worthy of notice, because we may reason conversely that if a country be flat, the beds are tolerably level, and extend some distance in any direction; but if it be much broken, that they have a high angle of inclination. Upon the dip other properties of the beds depend; and it will be seen that it involves many important consequences to works constructed on their outcrop. The relative elevation of varying deposits bears directly upon the flow of surface water from one area to another; therefore it affects the land springs, and the dryness or dampness of any given locality. This leads to the more extensive and intricate question of relative permeability, upon which depend the all-important points of the power of absorption of water by the beds, and the nature and origin of deep-seated springs. These points influence not merely the supply to artesian wells, but the liability to landslips, and the varying pressures by which engineering works are especially affected. The phenomena of deep-seated springs depend not altogether upon permeability—although this is one of the chief elements in their production—but also upon the relative position of pervious and impervious strata. These may succeed each other in the simplest way, by being in regular sequence the higher beds resting evenly upon the lower, each possessing the same angle of dip; or in a more complicated manner, which is described as "unconformable." This term is applied to beds, or to sets of beds, which at any particular spot possess different degress of inclination. It is evident that in such a case the uppermost beds rest upon the edges, and not upon the surface of those beneath, and that before the higher were deposited, the lower had been cut off by some process of denudation. Occasionally beds overlap each other, without being exactly unconformable; and sometimes those which are known to be so, do nevertheless rest evenly upon each other, and with the same dip; but this is a merely local, and may be considered an accidental occurrence. When strata are unconformable, of course the continuity of the lower beds beneath the surface is broken, and this must affect the flow of water through them in any direction. The underground extension of rocks is likely to be interrupted also by "faults," or other dislocations, by which portions of them are displaced, sometimes several hundred feet, and which may extend horizontally for a short distance only, or for several miles. Such faults or breaks must in all cases be discovered, and their influence estimated, in the consideration of problems in engineering geology.

(*c*) *The Bearing of the Nature* (*a*) *and Relation* (*b*) *of the Rocks upon Practical Works.*—The rocks which form the surface of the earth, whatever their nature may be in any particular locality, of course form the base of all engineering and architectural works. The same remark applies equally to mining and well-boring operations, and to agricultural pursuits; the rocks must exercise over them all a permanent influence, of necessity greatly conducive either to their success or to their failure; and this not only by their inherent properties or peculiarities, but also, as we

have seen, by the relations which they bear towards one another. These relations, and the broader characteristics of the strata, have been briefly mentioned above—*a* and *b*—and are more fully described further on. The ways in which their influence is exercised may be here enumerated, as indicating the necessity for all available geological information being secured, and calculations based thereupon, before works of any importance are commenced or even designed. This influence is exerted in two distinct ways, (*a*) Through the nature of the rocks; (*b*) Through their relation; but the two, although distinct, are sometimes blended, and always so intimately associated that a partial separation can only be attempted.

(*a*) The trial-holes usually made before the commencement of railways, and other engineering works, expose the kinds of rock near to the surface along a given line, or over a given area. But these holes, by themselves, afford no indication of the thickness of any deposit, and in this respect may actually mislead, unless other conditions be ascertained; a knowledge of which points may, however, be derived from the trial-holes, when once the method of utilizing their indications be understood. For instance, a line of such holes over a hill may be, almost without exception, in clay, one or perhaps two of them on the flank of the hill being sunk in a hard rock. The presumption would probably be that a cutting made through this hill must pass entirely through clay, except where the hard rock was exposed; the reality, that nearly the whole of the work has to be carried on through the harder stratum at a great additional cost. For the edge only of the bed was touched by the one or two holes where it comes to the surface, with a narrow outcrop, along the steepest part of the hill. This sort of thing has repeatedly happened, and more frequently still, a bed of gravel or sand, only a few feet in thickness, but spread over an extended area, or on the slope of a hill, has led to the conclusion that the whole of the cutting would be, as all the trial-holes were, in sand or gravel. It may turn out to be hard and intractable rock, removable only by blasting operations, but covered with gravel just thick enough to reach below the few feet exposed in the trial holes. Or the converse may be the case; the engineer unacquainted with geological methods, feels sure, from his trial holes, that he will obtain from such and such a cutting, building-stone sufficient for all the bridges, or gravel enough to ballast the line; but as the work progresses, and the deeper strata come into view, he meets with serious disappointment. These instances, selected from many, are sufficient to show the necessity for the nature of the rocks being ascertained, not merely at the surface, but to a depth below, certainly not less than that of the deepest cuttings.

If such information be necessary for cuttings, it is much more required for tunnelling through the rocks, where the work is far more costly, and where, consequently, much greater saving may be effected by a previous knowledge of what strata will, or will not, be passed through in any line at a given level. Yet here the indications from trial holes must be still more meager seeing that tunnels are seldom made except where the hills are too lofty to be passed through by open cuttings, therefore through strata at a greater distance from the surface. But the evidence so obtained, if treated by geological methods, may be made equally reliable, and in proportion far more valuable. Many tunnels have been made in places which would have been avoided had the geological phenomena been previously ascertained; others where, by diverting the line a short distance to one side or the other, they might have been made with a saving of more than half the cost of construction.

The nature of the rocks to be passed through in tunnels or cuttings affects also the calculations for the necessary slopes, and consequent widths, of embankments. For main-drainage works, trial holes afford ample information regarding the kind of strata along which the sewers are to be laid, but not as to their relation—a much more important point in this particular class of work, and one which is again referred to further on. The remark is applicable also to excavations for docks, foundations for dock walls and sills, water towers, and similar works. The evidences of the solidity of the rock, its liability to slip, or to squeeze outwards under pressure, being obtaina-

ble from trial holes and borings, need not here be considered.

In many cases the nature of the rock upon which bridges or culverts are to be built can be very well ascertained by trial holes; but not by any means in all.

For it happens that many of the largest and most important structures, such as bridges and viaducts, are required in the lower-lying parts of a district, that is, in its valleys; and here the evidence thus obtained is apt to be misleading. The smaller valleys are, in places, filled to a depth of many feet with a wash from the neighboring hills, composed perhaps of sand or clay; which wash so closely resembles the rock whence it has been derived, that it is, except to the experienced eye, a part of, and continuous with, the rock of which the high ground consists. But beneath this wash there may be a treacherous bed of peat, or even of quicksand, the evil influence of which is perhaps only discovered when the new structure is sufficiently advanced for its weight to cause an ugly settlement. In such places, it may be urged, borings are resorted to rather than trial holes; even where such is the case similar results may occur, should the misleading characters be repeated. And it often does happen that in alluvial flats there is a great number of rapidly alternating ancient river deposits, which may consist of peat, silt, gravel, sand, or clay. The solid substratum may not be reached perhaps for fifty, sixty, or even a hundred feet. if the spot in question be situated over an old course of the river, sometimes a long way from its present channel, and of which there is nothing on the marshy plain to indicate the existence.

A very important matter, in regard to the cost of construction in all engineering and building works, is the material which the rocks of the neighborhood will afford, and this of course varies according to the nature of the rocks themselves. The local quarries, lime-kilns, brick-yards, &c., will almost certainly be on the outcrop of beds most prolific in building materials. But a geological knowledge is nevertheless requisite to guide the engineer in laying out his works so that they may strike the more valuable strata to the best advantage. Building material is frequently brought long distances, when that which is as good, or even better, occurs—but perhaps hidden by a few feet of drift—in the vicinity, and, it may be, in abundance. A railway cutting or a tunnel may be judiciously set out so as to follow exactly the course of a useful stratum, even to a considerable depth from the surface, probably to rail-level. On the other hand, it may be planned so as to miss the bed, except just at the surface, or possibly altogether; for the point depends upon the direction of the dip of the stratum, its consequent strike, and the actual amount of its inclination.

The drift gravels occur in a more irregular manner than any other series of deposits, but if they be previously mapped, and the work be designed accordingly, a great saving may be effected; the labor of excavating a cutting, for instance, may perhaps be made to yield the additional result of affording ballast or road metalling. The extent and thickness of the gravels must be ascertained, also their mode of occurrence, whether as capping a ridge, filling an old channel, or resting on the sloping flank of a hill. Where gravels are scarce, or altogether absent, ballast may be obtained from the more solid rocks, broken up small for that purpose, and in some districts these are sufficiently plentiful. Even in many thick deposits of clay there are found occasional beds of hard septaria, or thin bands of limestone, suitable for the purpose; the line of these, when their outcrop has been traced, may frequently be followed with advantage.

Nearly, if not quite all the geological formations, yield some one or more forms of building material, and many consist almost entirely of rocks that can be utilized in construction. The varieties and qualities are numerous, but nearly all fall under the general terms limestone, sandstone, and brick-earth, and under these headings they will be dealt with further on, with reference to typical localities. But viewing the deposits on a large scale, one or two points may be noted that are worthy of remembrance. There are many series of strata which, being either all limestones, or all sandstones, or a mixture of both, with perhaps intervening clays, are grouped under some comprehensive term. These general in-

clusive denominations are convenient rather than strictly accurate; as instances may be mentioned the Lower Oolite limestone and the Caradoc sandstone. In every series there are some beds of more especial value for particular purposes than others above and below them, although the difference may not be at once apparent. There are beds also which for building works should be scrupulously avoided, in consequence of being liable to crumble on exposure, or possessing some other detrimental peculiarity of composition. As beds vary rapidly, that one which is good in every respect in one district being worthless in another, no general description can accurately apply to all localities; therefore these matters should receive careful and local investigation.

The minerals and metals which occur so abundantly in these islands, and the cost of mining for them, are also dependent directly upon the nature of the rocks with which they are associated, and the conditions by which those rocks have been affected since their deposition. In the most important instances they exist as an integral part of the formations in which they are found, as coal, ironstones, &c., but these are not, strictly speaking, either minerals or metals. Pure minerals and native metals are comparatively rare, but the terms are conveniently, if somewhat loosely, extended to include rock-masses, or portions of rock-masses, of which certain mineral matters, or metallic ores, form a characteristic part. The "minerals" and "metals" included in the terms thus qualified will be spoken of in another place, and for all practical purposes they may be treated in the same way as any other of the rocks in which they are enclosed, or with which they are interstratified. Their existence in any given area can be ascertained in a similar manner, their outcrop surveyed, their extent determined, and their value approximately estimated.

Agricultural pursuits are affected, to a degree, much greater than is perhaps generally understood, by the nature of the solid rocks beneath the surface soil, on which the success of farming operations is admitted to depend. For all soil, or mold, has been produced, during the lapse of many years, by the atmospheric disintegration of the surface of the strata which form the base or subsoil. It has been increased in depth and somewhat modified, but its constituents have not been materially altered, by the annual growth and decay of vegetable matter. The process has been assisted by the apparently trifling, but still ceaseless action of earth worms working into and turning up the subsoil, thus constantly adding new material of similar character. It is evident that the nature of the soils of any district must therefore vary as the subsoils or strata from which they have been derived, and in a corresponding degree; and that the geology of a place being known, the subsoils and soils are equally understood. A base of gravel or sand produces a light soil, abounding in silica, that substance not unfrequently forming four-fifths of its whole weight. This will vary from a fine sandy mold to a stony soil, as the particles of the rock beneath are fine and uniform in size, or coarse and irregular. Clay gives rise to a stiff, heavy, and sometimes tenacious soil, containing from 10 to 30 per cent. of alumina, and varying in quality perhaps more than any other kind, but being as a rule more productive. Limestones produce light soils, variable, and sometimes full of detached lumps of the rock, but generally yielding good returns for high cultivation.

There are certain natural causes which modify, to some extent, the normal characters of the soils thus derived from subærial disintegration of the rocks of a locality. The result of the influence exerted by these causes may not be extensive, but they are locally important. First the rain-wash, which removes the lighter particles of the rocks from higher to lower ground. In a flat country, the effects of this action are not very perceptible, but where the surface is broken by hills and small valleys, accumulations of this material may often be seen several feet in thickness. Then, where the downward progress of rain-wash has been arrested by a wall or fence on a hill-side, the result is, after a time, very evident, in the ground being unduly higher on the upper side. Again, the growth of peat and the accumulation of marsh-clay are agencies which give rise to soils very different from what they

would otherwise have been at the spots beneath such influences. The proportion of decaying vegetable matter, so useful a constituent generally, is in peat so excessive as to render the soil almost worthless for purposes of cultivation. The soil of which a marshy plain is composed is due to the rocks somewhat further up the valley, the particles having been brought down in suspension by the waters and deposited, thus forming a flat, when the stream has at times overflowed its banks. Further, where two indifferent soils meet, which have been formed from decomposition of contiguous rocks, that which occurs along the line of junction is generally found to be of better quality, owing to admixture. Some soils are rich in fattening properties and excellent for grazing, but through want of lime, without which no bone can be formed, young stock do not thrive upon their produce. The deficiency can often be supplied at a small cost, and the value of the land be thereby much enhanced; this is frequently the case in low marshy situations. Other soils, having had much of their productive properties removed by excessive croppings, may be considerably renovated by a surface dressing of the parent subsoil. The same object may be effected by deeper ploughing, the subsoil being of course gradually incorporated with the surface mould. But for this and similar operations, certain particulars must be obtained, or the labor so expended will perhaps have been thrown away. These are the chemical composition of the subsoil, and the constituents required to be added to the soil itself; details which are readily obtainable through a small expenditure of time or money by chemical, but not necessarily quantitative, analysis.

As the subsoil varies, so does the necessity for draining the land, and the facility with which the operation can be performed. Land-drainage, as ordinarily understood, is a simple matter, but there are some geological considerations respecting it to which attention may be briefly directed. As the strata affect the soils and sub-soils, so they must of necessity exert an influence upon the natural drainage, and upon the means best adapted to that of an artificial character. The springs of one locality being but the natural outlet of water from another, the strata that now throw out springs would, if occurring at a different level, act as the channels for draining water away from the surface to the interior, to be afterwards thrown out elsewhere. And it sometimes happens that the subsoils, or underlying strata, may be by some artificial aid, made available for purposes of drainage where they would not so act without that assistance. In other words, a plan of combined natural and artificial drainage can sometimes be easily carried out, where a natural system does not exist, and where an entirely artificial scheme can be adopted only with very considerable trouble and expense.

Although the last few years have witnessed a great, and, on the whole, beneficial change in the methods of disposal of town-sewage, the difficult problem is still far from being solved. In many cases plans of irrigation have been adopted, and these are always affected by the nature of the rocks upon which the sewage farms are situated. Opinion is greatly divided, not only as to the respective merits of the methods of precipitation and irrigation, but also, when the latter plan is in question, regarding the kind of soil best suited to the purpose. Doubtless much may be said on behalf both of the light and heavy soils, of the gravels and the clays, but a point that should not be lost sight of is the ultimate disposition of the water holding sewage particles in solution and suspension. A heavy soil will frequently yield enormous crops when judiciously irrigated and in favorable seasons, but beyond the mechanical deposition of its suspended particles, the water is not clarified; it runs off the land almost as chemically impure as when pumped or discharged from the reservoir. Very little of the liquid percolates down into a typical clay, and the benefit derived by the crops seems to be mainly owing to the moistening of the surface when it would otherwise be dry and parched. It is probable that pure water would have almost as good an effect as liquid sewage upon heavy land. Light soils, on the other hand, absorb and, for a while, retain a great deal of the moisture, giving it out again to the crops in a more equable manner; they may yield less produce, but this is, in some measure, through their containing within themselves a

smaller quantity of the elements of fertility. It seems reasonable to suppose that as the liquid is considerably filtered by its passage over or through sandy soils, the ingredients removed from it are ready for absorption by vegetation. A light soil, owing to its permeability, will at all times take more sewage liquid, acre for acre, than a heavy one, and in some seasons—during a period of floods, for example—this property may prove of great advantage. But considering the phenomenon referred to in connection with land drainage, that the springs of one part are but the drainage of another, it is questionable how far we are justified in saturating the sands of any locality with tainted water.

There are, of course, such soils as loam, intermediate between sand and clay, combining the characters of both in proportion to the quantity of each which occurs in their composition; and this kind of soil may ultimately be found the best for sewage irrigation. Or, what is more probable, an area partly on impervious clay and partly on pervious sand or gravel will offer the greatest advantages. For in addition to being somewhat more independent of the seasons, farms so situated would admit of the water, after running over the clay and there depositing its suspended matter, passing, by gravitation if possible on to the pervious beds. In its passage over or through these it would be more or less filtered, and the effluent water thus rendered, perhaps, sufficiently pure to be allowed to flow into a stream or river with impunity.

It has been shown that the "minerals" and "metals" for which mining is carried on depend upon the nature of the rocks with which they are associated. Upon this depends also in a great measure the kind and the cost of preliminary borings, of the main shafts or pits, and of the actual mining operations; but much more are these influenced by the relation of the rocks in and beneath which these are performed. For instance, "in no respect do collieries differ more from each other than in the quantities of water which they encounter, either in the mining or in the subsequent working of their mineral. In one case a retentive clay cover may prevent the access of surface water which in another may pass in abundance through a sandy or a gravel alluvium. In certain districts water-bearing measures of an almost fluid consistency must be passed through, whilst in others the comparatively tight coal measures may at once be entered. Frequently the strata above and below the coal are so compact as to render the workings actually too dusty and dry; but instances are common enough in which water makes its way through the roof stone, or through the coal itself, and adds difficulties and expense to the whole of the operations. When the measures through which the pit is sunk consist of stony rock, they are often allowed to stand open, but when shales preponderate it has to be walled with brick or stone, to which in some cases, as against the influx of water, wood or cast iron may be preferred. But when the measures are covered by other and more absorbent strata, saturated with water, the winning of a colliery becomes a most serious undertaking, tasking the energies of the best men, and sometimes collapsing after a ruinous outlay. Examples of these difficulties are afforded by surface beds of sand and gravel, and by the well-known red sand under the Magnesian Limestone. One of the most serious questions to be solved by the coal-viewer in the very outset is the system by which he means to work his mineral; and in order to form a judgment upon this head it is important that he should not only be acquainted with the various modes in use elsewhere, but should have acquired a knowledge of the peculiarities of the seams in his own district. Where the beds have a definite dip in one direction, the working pits are usually placed as far towards the deep as it is convenient to go, so that underground the coal may be brought down hill to the pit-bottom. Should the strata lie in a trough, the pits may advantageously be placed in its middle line, so as to command the coal on both sides."—See "Coal and Coal Mining," Smyth, 1872.

The relation of two or more rocks to each other may also affect engineering works even to a greater degree than the actual quality of the rocks on which such works are situated. This relation consists in their respective dips, their position in regard to each other, their

various qualities of permeability, and so on. In nothing is it more evident than in railway works that the relation of two beds—to take a simple case—differing from each other in kind, but the same in dip, may be such as to increase the cost of any work upon them, if risk to its stability is to be avoided. Let us suppose a railway cutting of moderate depth to traverse two beds of different character, one a water-bearing sand resting evenly upon a tenacious clay, both dipping at an angle of 3 deg. The slope on the higher side of the cutting will be scored by a series of weeping springs along the line of junction, which will surely, although perhaps slowly, cause serious slips unless means be taken for their prevention. Should the dip lie in the same direction as the fall of the ground—which is, however, unusual—the flow of water will be quicker at some times than at others, perhaps intermittent. Any structures, such as bridges over the cutting, must then have their footings well down into the clay, not on it, for its surface, even many feet from the ground-level and under an equal thickness of solid-looking rock, would be absolutely unsafe as a foundation. If the above simple instance demands precautionary measures, much more must the frequently intricate relations of the rocks receive careful consideration. The beds may dip rapidly, the water-bearing strata may be numerous, and the geological structure may be complicated by faults or by local unconformity.

In tunneling, a previous knowledge of the relations of strata to each other is still more desirable; indeed, it would be impossible to insist too strongly on the necessity for all geological details being known in regard to a hill to be pierced in that manner. Not only might the style of working be varied, but the form or strength of the tunnel itself might perhaps be altered with advantage, to suit either the varying pressure or the peculiarities of rocks known to occur in the center, different to those exposed in the cuttings at either end. Even the gradients might require to be modified according to the existence or non-existence of faults or of springs in the body of the hill, which if not previously detected would be discovered when too late to make any alteration. If the surface of a hill be carefully examined, the boundary lines of the beds of which it is composed be accurately surveyed, their dip ascertained, and the lines of all faults laid down, the position of the rocks within the hill—and consequently the points at which they will be met with—can be accurately determined. Even when the all-important point of the amount of dip cannot be obtained from actual sections, it can be worked out—by a method to be hereafter explained—from the boundaries, or other definite lines, if these have been laid down on the plan with precision. The highest of the beds in a series passed through by a tunnel generally occur near the center of the hill, unless they are inclined in one direction only. This is owing not merely to the fact of the rail level rising from each end towards the interior, but to a well-known geological phenomenon. As a general rule beds dip from each side into a hill or ridge; the statement being limited to hills and ridges as such, and not to include escarpments. Therefore, as the tunnel proceeds, beds are pierced higher and higher in the series, until the uppermost of all met with is somewhere near the middle.

Banks.—At first sight the relation of the rocks beneath the surface may not seem to have any direct bearing upon railway embankments, and similar artificial accumulations of material. But there are ways in which it does now and then greatly affect the cost of such works, and, what is equally important, that of bridges and culverts erected beneath them. A line of railway does not usually run in the direction of dip of the strata, but rather at right angles thereto, nearer to that of the strike of the beds, as it follows the contour of the country. The dip, whether great or small, therefore is generally away from the railway, either to the right hand or to the left as the case may be. If it be of any amount, say exceeding 5 deg., the bank, when it attains to any height, will be very likely to force the beds beneath it over each other along the planes of bedding, and thus give rise to slips, sometimes of great extent and involving much loss of material. These slips are usually sudden and liable to repetition, causing great expenditure for piling and other preventive measures. But if the

liability to slip, owing to the dip of the beds—which usually are, in such cases, alternating clays and dis-similar deposits—be previously entertained, it may be minimised by "running the tip ahead," and working backwards with the bulk of the material. Sometimes lines of railway follow the direction of dip for a short distance; if this be into the hill whence the material comes, there is no risk of slipping, but it is occasionally the other way, when slips are sure to occur. If these slips be quite forward they are sometimes of slight consequence, as the bank is then advancing in the right direction. But should any bridge or culvert have been built in their path, ready for backing up, the consequences may be, and frequently are, serious, for such structures are then almost sure to be overthrown, unless the bulk of the work be done in the backward manner mentioned above.

The preceding remarks apply equally to the construction of water reservoirs and canals, but it may be added, in regard to them, that the geological structure of a country must greatly affect the supply of water to canal-feeders and its retention in natural reservoirs. The "head" of water that can be maintained in such places, generally secured by a dam across some minor valley, and supplemented by pumping, is limited by the springs which may occur within its area; not by the amount they are capable of yielding, but through other phenomena yet to be described, which may, and frequently do, render utterly useless large expenditure for pumping. The attempt is, in fact, made to obtain a head of water which the geological conditions, left to themselves, make absolutely impossible; therefore, if these be understood, and measures taken accordingly, much cost and useless trouble may often be saved.

A smaller, but not unimportant matter, is the difficulty sometimes experienced in carrying out main-drainage works, owing to the surface springs and others tapped by the excavations. Several instances have occurred where, owing to the quantity of water thus met with, the plans, after commencement, have had to be altered, at great disadvantage; others, also, in which the pipes have been, through this difficulty, improperly laid, or have afterwards settled in the sands, so that the joints have ever after been imperfect. In consequence, they have admitted the spring waters, thus adding several hundred pounds a year to the cost of pumping, besides deteriorating the value of the sewage for irrigation or precipitation. These results have happened, in some cases beyond hope of remedy, not because the geological details, including a knowledge of the springs, could not have been ascertained in time, but simply because they have been ignored. With the remedy for such a state of things, where one is practicable, we shall deal in a succeeding impression, but it must always be adopted within certain limitations, and indeed, should only be allowed under official supervision.

One observation may be made with regard to foundations, whether of dock works, bridges, or buildings; it is that however solid the stratum may apparently be in which the excavations are made for foundations, the calculations as to stability are incomplete and liable to error—as the works are to unforeseen catastrophe—unless all the relations as well as the nature of the rocks beneath have been ascertained and taken into consideration.

An improvement in the water supply of a district is one of the practical results that may be expected to arise from the working out of its geological structure—that is, from the knowledge of its rocks and of their relation to each other. The supply of water in any given locality is not by any means proportionate to its rainfall; for the widely-spread water-bearing beds are great distributors, and by them it is to a great extent equalized. The supply to be obtained by boring down to deep-seated springs is practically inexhaustible, being scarcely, if at all, affected by drought, and these springs form the only source on which can be placed a full reliance. The phenomena of springs, and of the sources of supply to artesian wells, both of which are practically important, are entirely dependent on stratigraphical and physical features. An explanation of them will be given, as well as of the reason why sometimes salt waters occur far inland, and fresh water springs beneath the sea; why some waters are

chemically pure, whilst others are saturated with mineral salts.

Another point affected by similar conditions is the dampness of a locality, which sometimes renders almost uninhabitable what would otherwise be a desirable and healthy situation. This may arise either from the physical conditions of elevation, situation, rainfall, and so on, or from those of a purely geological nature. It is frequently owing to the saturated condition of the water-bearing beds immediately beneath, and probably close to the surface; in other words, by the proximity to the ground level of the general water line of the district. This is a question that should influence the choice of situation for all public buildings, and, indeed, for private houses also, where circumstances are such as to admit of selection. This will be fully considered in an article on the subject of "Sites," as one which merits, but seldom receives, much attention.

A knowledge of the undoubted relation existing between subsoil and disease must also be beneficial to those who are seeking for themselves a new home. The whole question cannot be entered on here, but the results of certain official inquries into one branch of the subject are appended. In the "Report of the Medical Officer of the Privy Council," for 1867, pp. 14-17, and 57-110, is discussed, from several points of view, the interesting question of the connection between the geological structure and the consumption death-rate of a district. After careful consideration of all the facts and statistics adduced, the following suggestive and valuable conclusions were arrived at, and may be considered as fairly well established. (*a*) That on pervious soils there is less consumption than on impervious soils. (*b*) That on high-lying pervious soils there is less consumption than on low-lying pervious soils. (*c*) That on sloping impervious soils there is less consumption than on flat impervious soils. (*d*) These inferences must be put along with the other fact, that artificial removal of subsoil water, alone, of various sanitary works, has largely decreased consumption. From which follows the general inference, that wetness of soil is a great cause of consumption. If this one disease can be so influenced that its ravages in a district may be, as they have been, lessened one half by a simple draining of the land, it may reasonably be assumed, that the power of other diseases also is more or less dependent on certain physical conditions, which are susceptible of natural or artificial modification.

ENGINEERING AND ART.

By Mr. CHARLES H. DRIVER, F.R.I.B.A.

From "The Builder."

"ENGINEERING," as the word is commonly understood, may be considered to be the science of "construction," and an engineer is likewise understood to be one who practically applies the theory and science of construction to the every-day wants and requirements of our lives.

"Engineering" is a very comprehensive word, including, rather widely, all matters relating to the formation of roads, bridges, canals, docks, harbors, lighthouses, mines, drainage, waterworks, sewers, fortifications, building, machinery in general, &c. Properly speaking, engineering is divided into two classes, viz., civil and mechanical engineering. In the first are comprised road-making, bridge-building, canals, docks, harbors, waterworks, drainage, mines, &c., and all the other works which may be classed under the head of railway, hydraulic, and mining engineering; whilst the second is principally connected with the manufacture and use of machinery, the working of metals, the construction of railway plant, steamships, guns, armor, plates, &c. In works of the first-class the "contractor" plays an important part, as it is he who executes the work from the designs of the engineer, and on his ability and good management the success of many undertakings very materially depends. In the second class, however, the case is somewhat different, as the

mechanical engineer is very generally both designer and executant of the work he undertakes. Thus much for the engineering of the present day, which may be said to have taken its stand, as a distinct profession in England, about the middle of the last century, and has since, by its varied achievements, done so much for the world at large,—much for comfort, much for luxury, much for wealth, but little, alas! for "art." All must admire the wondrous powers of such magicians as Watt, Telford, the Stephensons, Brunel, &c., and all will admit that they and their works have wrought an enormous amount of good; yet we must admit that though their works contain a vast amount of the *utile*, they have little of the *dulce* about them. Such, however, was not the case with the engineers of antiquity, who not only constructed works of pure utility, such as the harbors of the Phœnicians and of ancient Greece, the bridges of boats make by Xerxes to transport his army into Europe, the canal across the isthmus of the peninsula of Mount Athos, the aqueducts, roads, and bridges of ancient Rome, but they constructed, or assisted in the construction of works which are now looked upon as the triumphs of art (architecturally), such as the temples and pyramids of ancient Egypt, the Palaces of Nineveh, the splendid buildings of Greece and Rome, and in latter times the glorious cathedrals of Europe and of our own country.

"Art" is the result of the endeavor of the human mind to achieve the perfection of beauty, whether it be in form, color, or sound, and, like engineering, it is universal; but, unlike engineering, it is strictly confined to man, and is the result of that power of selection that man, of all animals, alone possesses. Many animals, birds, and insects execute work which rivals and excels the best and most delicate work of man, but which, although artistic in result as to form or color, is not art in itself, and birds or insects cannot be credited with "*art*," as they only follow a line of action they have no will or power to alter. The word "art," as generally understood, is applied to all such matters as music, painting, sculpture, architecture, medicine, agriculture, &c., and may be divided into two classes,—viz., "fine art" and "useful art." Music, painting, sculpture, architecture, &c., are specifically termed fine art. Medicine, agriculture, &c., belong to useful art. It is difficult, however, to define exactly where "fine art" ends and "useful art" begins, the two being, as they ought to be, so closely united. "Art," in the abstract, may be considered to be that which gives pleasure to the purely mental faculties as opposed to the purely animal passions. In this sense we view and accept the mental pleasures afforded to us by music, painting, sculpture, or literature, in contrast to the bodily pleasure we obtain by warmth or coolness, eating and drinking, rest, &c.

Again, the pleasures afforded by art are not confined to individuals or nations, but are universal, whilst those which are corporeal are for the most part personal and selfish. The beauties of nature, such as are seen in fine landscapes, glowing sunsets, the flowers, the songs of birds, &c., yield pleasure to all, and of them there is no monopoly, and therefore they belong, in the truest sense, to art; and it is in this sense, and with this aim, viz., that of giving the greatest amount of pleasure to all, that the painter, the poet, the musician, and the architect should work. The mental pleasures which are embodied in the word "art" reach us chiefly through the eye and ear, the organs of smelling, tasting, and feeling ministering more to our bodily pleasures. Of the two which we may call the artistic senses, viz., seeing and hearing, the one which now concerns us at present is that of "seeing," for it is by means of sight that we learn to appreciate those attributes of "art" that are most nearly connected with works of construction, viz., "sublimity," beauty, grace, harmony, picturesqueness, proportion, order, and fitness; as, for example, the agreeable effect designated by "fitness" is an artistic pleasure which may be called the æsthetic of the useful; as, when a work is not only done effectually, but done with the appearance of ease, or the total absence of restraint, difficulty, and pain, we experience a delight quite different from the mere satisfaction growing out of the end obtained. Much of the pleasure of architectural support is referable to this source. Among the pleasures that are afforded by artistic arrangements may be noticed the sense of "unity

in multitude" arising when a great number of things are brought under a comprehensive design, as when a row of pillars is crowned by a pediment. The use of simple figures,—the triangle, circle, square, &c.,—for enclosing and arranging a host of individual parts has a tendency to make an easily apprehended whole out of a numerous host of particulars. In all large works abounding in detail, we crave for some such comprehensive plan whereby we may retain the total while surveying the parts. A building, a poem, a dissertation, or a speech should have a discernible principle of order throughout, the discernment of which gives an artictic pleasure even to works of pure utility.

Let us now proceed to consider my third point, viz., are engineering and art of service to each other, and can they be united? I think, perhaps, it will be as well to sub-divide the question by seeing, firstly, whether engineering and art are of service to each other; and, secondly, can they be united? Let us, then, begin by considering how art is benefited by engineering. Engineering benefits art when engineering is in itself good and right, and when it does not benefit art, it is because it has been wrongfully and improperly applied, and I must ask you to take this remark at what it is worth, for time will not permit me to go fully into the point; but, as an instance of the good that accrues to "art" from engineering, I may refer to printing, and its attendent belongings. How much does art owe to it? Truly, in the days when printing was unknown, there were then, as now, poets and philosophers who expressed their thoughts in poetic language, and chronicled in strong prose the actions of their fellow men. But to how limited a number was the pleasure that their works afforded confined, as compared with the countless thousands who now delight in their genius? Then every copy had to be painfully transcribed, each copy taking months, perhaps, to complete. Now, by the aid of engineering (machinery), hundreds of copies can be supplied daily. Engraving, again, is an art that has greatly benefited by engineering science; for when it was, as it used to be, confined to copper plates, comparatively few copies could be produced. The copies thus produced were necessarily so costly, that only the wealthy could obtain them. Now, by using steel plates and the electrotype process, the copies are so multiplied, and thereby cheapened, that they are within the reach of all. Engineering affords great facilities for reproducing beautiful form and material, as, for instance, in such matters as porcelain and glass ware, iron and bronze work, textile fabrics, &c., thereby affording pleasure to many. But it may be,—as it has been said,—that all this tends to vulgarize art, and that machinery produced articles, however beautiful, are not artistic, *because* they have been produced by machinery. This argument (I speak with all deference) is, I think, very fallacious. If a vase or a cup is in itself artistic in design, and is good art, so are equally so the 20,000 copies of it, provided they are exact copies, such as machinery has the power of making. And, again, a gardener produces from a chance seed, or, by care in cultivation, a flower or plant that has some especial point of beauty, and for which he obtains a very large price. By-and-by, from cuttings or other means of propagation, the plant or flower becomes common, and can be bought for as many pence as it cost pounds before. The plant or flower has not changed, it has just the same beauty as it ever had, but instead of pleasing only a few it pleases many. And it is on the ground that they give mental pleasure to many, that I claim for the multiplied copies of an artistic original the right to be themselves considered artistic. Engineering itself, however, is not artistic, nor does it directly produce art, but it *does* disseminate it, and, therefore, engineers may very fairly be said to benefit art.

But now let us consider how art reciprocates the benefits she receives from Engineering. In the earlier days of engineering, art certainly was of great service to engineering by teaching those who practiced it to clothe their works with beauty; it taught them, when they wanted a vertical support, to give it a tapering form, to give the gentle swell of the entasis to the column, thus satisfying the eye's sense of beauty and grace; to build the temple, the palace, and the cathedral all with proper fitness, and all with beauty. Though engineering and art are, and have been, co-existent, art is the master, directing and guiding engi-

neering into the right paths; and while engineering acknowledges this mastership of art all is well; but when, as is the case in these modern times, engineering strives to obtain the mastery, the result is chaos as regards art. Modern engineering no longer pays allegiance to art, and arrogantly considers that it can do quite well without it, and hence it is that we have gone on with an increasing loss of beauty in the works of our modern engineering (this remark, however applying only to the civil, and not to the mechanical engineer). The modern engineer seems utterly to ignore beauty of form; if he has to use a vertical support, he will probably make it of the same thickness all the way up; if he wants a buttress, he will put a great lump of masonry or brickwork in, without thought as to whether it will look ill or well, but calculated, I grant you, to do to an ounce what it has to do. There is an immense amount of thought in their work, but no mind,—I mean in an artistic sense; and even the thought they give to their work is of a sordid kind; for while they study how to use their material economically, they omit at the same time to study how to make their work pleasing, thus leaving half their work undone. I admit that engineering possesses a strong point of affinity to art in its *truth*, and by this I mean the honest construction employed by engineers in their work, never disguising or hiding it, but letting it be plainly visible to all; and there is a good honest purpose in what they do, and the sentiment of reality and truth, as opposed to fiction and falsehood, appealing as it does to our practical urgencies, disposes us to assign a high value to every work in which truth is strongly aimed at, and to derive an additional satisfaction in work in which fidelity of rendering is induced upon the charms peculiar to art. But while we admit and admire the truth of the engineers' work, and give them their due meed of praise for what they do, we must not forget at the same time to blame them, in that they leave so important a part of their work undone; viz., the making of it artistic. To my mind it is very nearly of equal importance that a building, a bridge, or whatever it may be, should be of good form and pleasing to the eye, as to be strong. There is an impertinence and brutality and want of regard for the feelings of others in many of the erections of late years,—the work of modern engineers. Consider how the Thames at London has been maltreated, and I ask you, as reasonable men, what right have we to inflict on ourselves and future generations such awful examples of the selfish disregard of all that is beautiful as have there been perpetrated? Our forefathers left us, their successors, works of beauty. What shall we leave our successors?

What remedy is there for this? Can Engineering and art be united, and if so, how? My answer is, yes, and by means of Architecture; for though, as I have said, Architecture is to be considered as the child of Engineering, yet it is through the graces of that child that we must hope to again reconcile and unite Art with engineering. I say "again reconcile," for in old days Art and Engineering were united, and Architecture was their offspring. We moderns have divorced them; let us re-unite them. But how will Architecture unite the two? Engineering is the science of construction. Architecture the art. Engineer is the hard matter-of-fact, uneducated man of business. Architect is the cultured and polished gentleman. Architecture is educated engineering. A man may be honest, bold, truthful, and business-like, but hard, selfish, and inconsiderate, with no knowledge or care for the beautiful. He is the modern engineer. Another may be honest, but timorous, uncertain, and unbusiness-like, kindly considerate with respect to the feelings of others, and with a great desire for the beautiful, often failing from lack of power. He is the modern architect.

Perhaps you will think I do not draw a flattering picture of the representatives and practisers of that art by which, as I have already told you, art and engineering can be united. Well, that may be true, but it is only true as regards the modern architect, as my first picture is of the modern engineer. The modern architect is as the times, his own want of pluck, and the modern engineer have made him; he, if I may say so, has been bounced and bullied out of his proper position by the engineer; and much that of late years has been done by engineers should have been done, and would have been better done, by architects, if they

had but retained their proper position by keeping themselves abreast of the times. I have given you a figurative sketch of the modern engineer and the modern architect. I will give you one as to the past. Here we have a man who is honest, bold, and truthful, certain and business-like, with kindly regard for the feelings of others, an intense love and knowledge of the beautiful, with a desire that all should participate in the pleasures it affords, and having the will to do good work, has also the power to execute it; he is the architect and engineer of old, the architect and engineer being one. . . .

The difference in the way an engineer and an architect would set about the same piece of work, if they had it to do, would, I think, be as follows, viz:—The engineer would begin by studying the purpose of the intended structure, then consider as to the best materials of which to erect it, and calculate to a nicety how little of each material he could use. If he wants a door, he puts in one; if he wants a window, a window is put in; but he never studies or thinks for one moment whether he can, by placing his door or window in some other position, improve the appearance without altering the utility of his work, and the result is, his work is as bald and ugly as possible. He then, perhaps, proceeds to what he calls "ornament" it, by sticking on here and there a moulding, which as likely as not he places upside down, thus destroying the only grace his work possesses, viz., truth; and then says, with satisfaction, "See what we engineers can do: we want no architects with styles and orders to teach us what to do." On the other hand, the architect (I speak of the true architect) would first, like the engineer, consider well the purpose of his intended building, and, like him, consider as to the best material to build with, and how best to use it. He also would place his doors and windows in their most useful positions; but, in addition, he would, from the first commencement of his work, have in mental view before him a certain effect, which he would strive to produce, and thus, while so planning his building as best to suit the requirements of the work, he at the same time makes it pleasing and artistic; and when the work is finished, it needs not the extraneous aid of moulding or applied ornament, for all that is wanted in that way forms part of the structure itself. It is common to find engineers twitting architects with their want of knowledge of scientific construction; but I think I can safely say of architects that they, as a body, have a far greater knowledge of practical engineering than engineers have of architecture,—as their works show,—the architect's failures in "construction" being as nothing to the engineer's failures in "art."

THE INJURIOUS EFFECTS OF THE AIR OF LARGE TOWNS ON ANIMAL AND VEGETABLE LIFE, AND METHODS PROPOSED FOR SECURING A SALUBRIOUS AIR.

By WILLIAM THOMSON, F.R.S.E., F.C.S.

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In considering these questions it is important that we should have clear ideas—first, as to what is pure air; secondly, for comparison, what alteration in the constituent parts of air would make it incapable of supporting life (say, for instance, air in which a candle could not burn); and, thirdly, between this wide gulf, what may be termed impure or unhealthy air.

Air is composed principally of a mixture of two gases, oxygen and nitrogen, in the proportion of about 21 volumes of the former, and 79 volumes of the latter. Oxygen is the constituent which supports life and combustion; nitrogen is an inert gas, which was doubtless intended by the Creator to dilute the active, or life-and-combustion sustaining-gas, oxygen.

The following may be regarded as an analysis of the purest air which can be obtained:

	Per cent. by volume.
Oxygen	20.999
Nitrogen	77.568
Carbonic acid gas	.033
Water vapor	1.400
	100.000

Dr. Angus Smith found that if he abstracted about 2½ per cent. of oxygen from pure air, it would not support the combustion of a candle and it would be impossible for any one to live long in such an atmosphere; so that the air, so far as life is concerned, may be compared to a reservoir with pipes placed near the top—the water under the level of the pipes being useless, except as affording a support for the top stratum, which can be utilized by being drawn off; so we have 18.5 per cent. of the oxygen of the air of no greater value, as a life supporter, than the water which lies under the level of the pipes of our assumed reservoir, and we may take it, therefore, that only one-eighth part of the volume of the oxygen of the air is of practical value.

We must, then, look upon air which has even a very minute fraction less of oxygen than that given in the above analysis, as more or less objectionable or bad, because the part of the total amount of oxygen contained in the atmosphere available for supporting life is so small.

The following analysis made by Dr. Angus Smith, shows the actual proportions of oxygen contained in air from different places, the last four of which cannot be regarded as otherwise than unhealthy:

	Per cent. by volume of oxygen.
Tops of hills (Scotland)	20.980
In the outer circle of Manchester (not raining)	20.947
Low parts of Perth	20.935
In a sitting-room which felt close but not excessively so	20.890
Pit of a theatre, 11.30 p. m.	20.740
About backs of houses and closets	20.700

It will be observed, then, that judging from the deficiency of oxygen alone in the air from the pit of a theatre, for instance, a draught of over 10 per cent. has been made upon the total available oxygen. Judging, however, from this point of view, we are assuming that the place of the oxygen abstracted is filled by an innocuous gas, such as nitrogen, which is not the case. When the oxygen is consumed by man, it combines with the carbon from his lungs, forming carbonic acid gas, which may be regarded as a positive poison, and if we take the amount of this noxious gas found in a theatre, we see by Dr. Smith's analysis, that it amounts to 0.320 per cent., being 10 times greater than that found in pure air. In Manchester streets, in ordinary weather, the air was found to contain 0.403 per cent. of carbonic acid gas, and although this is a very small fraction, it is yet about 20 per cent. greater than that found in pure air.

When we consider that a healthy man will pass through his lungs about 2,000 gallons of air per day of 24 hours, we see that he will during that time inhale 5¼ pints of carbonic acid gas in the country, and 6½ pints in the streets of Manchester, and about 51 pints in an atmosphere such as that found in the pit of a theatre.

These, however, together with organic emanations, are the impurities which are found in air polluted by the breathing of animals, but these are not the only impurities found in the air of large towns; others more irritating and poisonous proceed from the burning of coal, to which I will refer later on; but to confine ourselves to the physiological effects of air charged with large quantities of carbonic acid, such as that found in a theatre, I will refer to certain statistics. It is well known that children who are kept after birth in badly ventilated rooms, die in large numbers from what were termed "nine-day fits." In 1783, Dr. Joseph Clarke, master of the Rotunda Hospital in Dublin, recorded that during 25 years in that hospital, when the ventilation was bad, no fewer than 3,000 out of 18,000 children born there died within the first fortnight of their birth. The number of these cases at once rapidly diminished as soon as the better ventilation of the wards was attended to, so that, in the following 28 years, out of 15,072 children born, only 550 died, being 1 in every 104, instead of 1 in 6, as was the case previous to the better ventilation; and since, Dr. Mc Clintock, one of the successors of Dr. Clarke, reported in 1861, that further improvements having been made in ventilation, the disease was then almost unknown.

Again, in the Mediteranean squadron, a fearful lung disease made its appearance among the men in 1861, which was found to be due to the want of proper ventilation in the lower decks of the ships where the men slept. The lung is a most delicate organ, in which the blood meets with the air which we breathe. The air passes into the lungs, through air tubes, which divide and subdivide, like the stem and branches of a tree, each branchlet ending in a small air cell or leaf, which is formed of a delicate membrane, not more than the thickness of a soap bubble; and the lungs of a full-grown man contains six hundred millions of these leaflets or air-bags. The membrane of these delicate air-bags is covered with fine, hair-like blood vessels, through which the blood flows, and thus spreading over an enormous surface it becomes aërated. The blood from one side of the heart, charged with the impurities, etc., which it has received from the body, passes across the lungs through the tiny blood vessels on the delicate membrane, and there meets with the air, the oxygen contained in which burns or oxidizes the impurities in the blood, carrying them away as carbonic acid gas, which we exhale, along with the nitrogen and excess of oxygen. This delicate organ is protected from injury by a variety of means. If, for instance, a particle of solid matter or liquid, for instance, touches the air passages as is sometimes the case in eating or drinking when the liquid or food is said to "pass into the wrong throat," a reflex action is produced, involuntary coughing takes place, and the intruding matter is expelled. If, however, the particles of matter be not sufficiently large to produce violent coughing, it produces a rapid secretion of mucus from the mucus membrane of the air passages, in the same way as a particle of dust coming in contact with the eye produces a flow of tears, which eventually washes away the irritating particle. The mucus, or living membrane, or skin of the air passages, is also provided with another most ingenious appliance; it is covered with myriads of minute hairs or cilia, which are in constant motion, like a field of corn in the breeze, so that when the mucus secretion, which is a glairy fluid, has taken hold of the offending particle, the movement of the small hairs passes it slowly along, till it comes near to the top of the trachea, or wind-pipe, when the mixture of foreign matter and mucus may be finally dislodged by coughing.

The results of this process can be observed when we breathe some time in the open air during a fog in a large town. The air then contains a copious amount of soot in suspension, which finds its way into the air passages, and which is ultimately expelled mixed with phlegm, the whole being sometimes of a dark or even black color.

If, however, the air passages are constantly being irritated by small particles of foreign matter, the mucus membrane gradually loses the power of performing its normal functions, and becomes diseased, and this diseased condition soon extends to the lung itself. Remarkable examples are to be found of diseases originating directly from this cause in the men employed at certain manufactories, such as knife and fork grinders, where the fine particles of steel float in the air, and are inhaled by the men. The lives of the men engaged in such occupations are extraordinarily short, a knife or fork grinder being considered an old man at the age of thirty, and if he has worked at his trade from boyhood without using fan appliances to take away the steel dust, as most of them do, having a strong prejudice against any such apparatus, they often commence at the age of twenty-five or twenty-six to die of a long and painfull illness, resulting from lung disease.

This may serve as an example of how the lives of men may be much shortened, by the irritation resulting from inhaling minute particles of a hard and angular dust, but the minute particles of carbon which float in the air of our large towns, and which may be found adhering to the sides of the air passages of the nostrils, for instance, after one has spent some few hours in a town, assuredly have an injurious effect upon the health and lives of the inhabitants. The same kind of irritation may, however, be produced by the inhaling of noxious gases, which are also found in town air.

Many other cases of consumption being produced in strong, healthy men, by breathing impure air, may be given; but, as one more instance of the pernicious

effects of bad air, we may refer to the average life-time of the people living at different places, which will speak for itself.

In London, the average	life-time is	29	years.
" Manchester	"	21	"
" Surrey	"	34	"
" France	"	34	"
" England	"	29	"

Supposing a man to take pure air into his lungs, containing, as it does, about 21 per cent. of oxygen and $\frac{3}{100}$ths of a per cent. of carbonic acid gas when he exhales it, the carbonic acid gas would be found to have increased to 5 per cent., and the oxygen to have diminished to about 13 per cent. If he were compelled to breathe again the same air, death would ensue in a few minutes; this air must, therefore, be largely diluted with pure air, and different hygienic authorities hold that each volume of air exhaled should be mixed or diluted at once with from 130 to 200 volumes of pure air, and that whilst a healthy man cannot pass through his lungs more than about 72 gallons of air per hour, he should be supplied with, at least, 10,000 gallons, that the impurities exhaled may not prove poisonous or deleterious to him.

The above facts show us the vast necessity of being supplied with pure air, and yet, strange to say, this is one of the points which by the generality of men least attention is paid. Millions of money are being spent to obtain for our towns liberal supply of pure water, which, is no doubt, most important for our welfare, but of much less importance than the supply of pure air. What a lamentable sight is presented to us, when we go to the top of a hill near to one of our large towns and look down upon it. We observe that it is enveloped in one vast cloud of smoke, and yet in such atmospheres thousands are living, suffering, and prematurely dying; and still more marked to the eye is the injurious effects which such atmospheres have upon plant life. Few plants or trees can grow in or near our large towns, and those which do stand better these injurious influences become weaker and weaker every year, until they ultimately succumb.

We ask what is the cause of this, and the answer is undeniable. It is owing to the gases produced from the burning of coal. When coal is consumed, a large number of different products of combustion are formed, which escape up the chimney and enter the outside air. Coal is burned in large towns for two purposes, the one for the processes of manufacturing, the other for heating our rooms and for culinary purposes. Up to the present time, wealth has almost entirely ignored the nuisance produced from the house smoke, but has acted vigorously against that produced from manufactures; and although this is a step in the right direction, it is unfortunate that it has only attacked by far the lesser of the two nuisances.

The smoke or gases produced from coal used in manufacturing are of a different composition from those produced by coal burnt in the ordinary room or kitchen grates. In the former, the coal and its products are more completely burned than in the latter; that this is so may be observed by looking at a room-fire in comparison with the fire under a large boiler. The general mode of keeping up the fire in an ordinary room grate is to put some pieces of coal on to it when necessary, either as large pieces when the fire is intended to last for some considerable time, or as small pieces, or as a mixture of both, when a good fire is required; in any case we may observe, when this is done, that a distillation takes place, and a yellow vapor begins to flow slowly up the chimney, and later on these products become ignited; on the other hand, the gases evolved from the coal by distillation in the front are completely consumed in passing over the red-hot fuel in the back part of the furnace. The smoke, therefore, from domestic fires is of a more complicated nature, and more injurious to the health of men and animals than the thoroughly burned products which escape from furnace fires, and not only so, but the smoke from domestic fires is distributed all over our towns, and allowed to escape into the air generally not more than a few yards from the level of the streets, and it is wafted down by the wind or air currents, contaminating the air which we have to breathe, and making it look as if a faint haze existed everywhere in our town atmospheres; indeed, in my opinion, the sanitary conditions would not be materially improved if all the manufacturing industries at once ceased to exist. This

is borne out by a paper read before the Manchester Philosophical Society by Mr. Peter Spence in 1869, in which he describes the results of a series of experiments made at his own house with blue litmus paper. These papers were changed morning and evening for some time, until he became an expert judge of changes in color produced by the air blowing from different directions. He found, as a rule, that on Saturdays, when the manufactories were not working, more acidity was observed in the air than on any other day or night in the week.

The principal impurities found in the air of large towns are soot, hydrocarbons, sulphide of ammonium, carbonic acid, sulphurous acid, carbonic oxide, and probably very minute quantities of arsenic expelled from the pyrites existing in the coal. All these are due to coal smoke, others exist such as emanations from putrid or decomposing organic matter, but by far the greatest quantity of atmospheric impurities in large towns can be traced to the emanations from burning coal. The sulphur compounds contained in the different atmospheres is regarded by Dr. Angus Smith as a measure of the pollution of the air from coal smoke and other causes, such as putrefaction and decomposition, but principally from the former, and these compounds, such as sulphurous and sulphuric acids, &c., he has estimated; and, calculating the sulphur compounds into sulphuric acid, he has arranged the following interesting table of results. Taking the sulphates in the air found in Valentia, in Ireland, as 100, we have the following amounts found in various places throughout the United Kingdom:

Scotland.—Sea-coast country places, west	132.2
England.—Inland country places	202.2
Scotland.—Towns (Glasgow not included)	604.4
London, 1869	750.5
English towns	1255.3
Manchester, 1869	1526.
" 1870	1757.8
Glasgow	2591.

Again, Dr. Smith compares the amount of acidity, or free acid in the air of different places. The free acid, as a rule, comes directly from the burning of coal, and may be regarded specially as the plant-destroying ingredient of town air. He gives the amount of ammonia which is eliminated principally from the burning of coal, and the following gives some of these results. In the case of acidity, the air of Valentia contains none, and cannot, therefore, be taken as a standard of comparison; but, for ammonia, the amount found therein is taken as unity. Here follows the table:—

	Acidity.	Ammonia.
IRELAND.—Valentia	None.	1
SCOTLAND.—Sea-coast country places, west	1	2.69
ENGLAND.—Inland country places	None	5.94
London, 1869	27.97	19.17
Manchester, average of 1869 and 1870	73.44	35.94
Ditto, 1870	86.76	36.54
Glasgow	109.16	50.55

The question has often been raised as to whether black smoke did more damage to animals and plants than the products of the complete combustion of coal, and some held the view that black smoke was innocuous, whilst others held that it was highly deleterious.

It has been shown that if a plant be completely covered with pure carbon, the plant will grow luxuriantly, but it has also been shown that soot is not pure carbon, but is carbon saturated with sulphurous and sulphuric acids, tarry matters, ammonia salts, &c., and it has been equally clearly shown that soot or black smoke is highly destructive to plant life. The soot, or black smoke, no doubt acts injuriously on plants in two ways, the first mechanically, by the tarry ingredients interfering with the free action of the stomata, or breathing orifices of the leaves; the second chemically, by the acids, salts, and other substances which would be washed from the smoke deposited on the leaves by the dew or rain which comes in contact with it, these substances would be absorbed into the substance of the leaf, and, coming in contact with the juices of the plant, would decompose it, and so render it incapable of imparting nourishment. It is remarkable to observe that the leaves of plants and shrubs in large towns are generally so thickly covered with smoke, that, if they be rubbed between the fingers, the latter will be much soiled or completely blackened. The effect of the air of large towns is to prevent the early budding of the leaves of the trees and plants, and to make the leaves fall early

in the autumn, or before it sets in. It is clear, then, that it would be a great advantage, to both animal and plant life, if black smoke were not emitted from chimneys, because it falls rapidly on to the plants, bringing with it, and consequently acting as a vehicle for carrying, the really noxious ingredients of the smoke. It also falls on and blackens the magnificent public buildings which most English towns possess; and, not only so, but injures the stone and carving exposed to it, by reason of the dirt being washed from it into the stone, many kinds of which ultimately undergo a crumbling process from that cause. It is evident, then, that it is important that, if possible, coal should be burned thoroughly, and no black smoke produced, because, as I have mentioned, it injures, not only plant and animal life and buildings, but soils everything which the good housewife considers worthy of being kept free from stain. If, then, it be found that cleanliness can only be obtained or retained at an immense cost of labor, many will lose heart, and become contented to remain dirty; and so the emission of black smoke from chimneys should and does prove a great social evil. If, however, the smoke were completely consumed whenever coal is used, the evil would not be stamped out, because the rain which falls in large towns dissolves from the atmosphere the impurities which it brings down with it, and which prove highly deleterious or poisonous to the plant. It is calculated that about 1½ millions of tons of ammonia, or salts of ammonia, are dissolved per annum from the atmosphere by the rain which falls on the globe, and this furnishes the plants with a most valuable food—the ammonia being formed in nature by the electric discharge, or lightning, or by the decomposition of animals and plants. What a vast difference is presented by the rain water which falls in and near towns. If a vessel be exposed in a town the rain water falls into it, but so does the black smoke, and after a few days if the water and soot which has fallen be mixed together, the liquid will present an inky appearance, and will change the color of blue litmus to red, whilst a vessel left for the same length of time in a country place will be free from sooty or noxious ingredients.

Lastly, I may mention that it has been calculated that about two millions of pounds worth of property are destroyed in London and its vicinity alone per annum by the emanations from burning coal. I do not vouch for the accuracy or otherwise of the last calculation, but, doubtless, we have sufficient proof that an immense amount of property is annually destroyed by allowing coal smoke to enter our atmosphere, as it does at the present time in England; and this is one of the points which weighs heavily in the scale on one side to show that, even if some scheme were devised and worked, even at great cost, it is probable that, looked at from a commercial point of view, from depreciation by damage to property alone, the scheme might prove a success; but this is one of the least important items in the problem. One which stands much higher is the increased degree of health which even the strongest of town inhabitants would experience, even by a small mitigation of the amount of impurities thrown into the air. I think that any one who works in the vitiated atmospheres of towns must feel, if he has tried it, that he can do more work, and can do it more easily and comfortably, in the atmosphere of a country place than in that of a town, and further, that he requires less food when working in a pure atmosphere than when employed in a vitiated one, the wear and tear on the system being much greater in the latter.

If, then, it were possible to secure a fine atmosphere in towns, what a vast benefit would be conferred upon the inhabitants. It would tap thousands of springs of energy which at present are kept dormant, and that energy, if directed towards the development of the country would bring back to it much of the prosperity of which it at present so much needs; but, above all, it would give an increased supply of good health and spirit, and would prevent many of our unfortunate citizens from wasting their money and destroying themselves mentally, morally, and physically, by excessive indulgence in stimulants.

So much, then, for the benefits which would be gained by improved sanitary conditions of our atmospheres; and I come now to the question whether it be practicable to prevent much of the con-

taminations which at present find their way into our atmospheres, and I think there there can be no doubt that it is. Schemes have been discussed of draining away the noxious gases from our chimneys, and passing them along the sewers, and ultimately allowing them to escape into the atmosphere from the tops of tall chimneys, but nothing was brought forward of a definite nature till Mr. Peter Spence, F.C.S., of Manchester, the well-known manufacturing chemist, made the necessary calculations, and embodied his scheme in a paper read before the Manchester Philosophical Society, 22 years ago. To give his ideas in as few words as possible he says that it is practicable to build a chimney 600 feet high, 140 feet external diameter at the bottom, and 100 feet internal diameter at the top, at a cost of about £40,000, and calculating the amount of air required for the complete combustion of the whole of the coal burned in Manchester, assuming that that air would enter the tall chimney at a temperature of 100° Fah. (about the temperature of the human body), the column of air would ascend at the rate of about 40 feet per second, and would be capable of carrying away about ten times as much air as that required for the combustion of all the coal employed in Manchester.

We may, assume, however, that if such a chimney, were built, and subsequently found incapable of doing the necessary drainage work, it is possible that a large fan could be erected in connection with the chimney, and the draught so aided by mechanical power.

The scheme has many interesting points connected with it. First, it combines the liquid and gaseous sewage, and makes the one in many respects neutralize the other. The sulphurous acid, hydrocarbon, &c., from the coal smoke, would prevent the decomposition of the liquid sewage, whilst the liquid sewage would condense much of these noxious substances from the smoke which now contaminate our atmosphere. Again there would be a constant draught through the sewers, and, consequently, it would be impossible for any foul air to escape from them; on the contrary, if there should happen to be any leak in the sewers, the pure air would flow into them, instead of, as is the case at the present time, the foul air of the sewers finding its way out from every crack into our houses; because it seems almost practically impossible to keep sewers so tight that objectionable smells will not proceed from them—when one place is made tight the bad odors make themselves evident from another. Mr. Spence relates an interesting method by which he prevented the sewer gases from entering his house. He found that one of the bedrooms in a house which he had occupied for a short time could not be used, owing to the bad smells which evidently found their way through some cracks in the wall of the room, and he hit upon the plan of joining a 4-inch iron pipe with the sewer at one end, and placing the other end a few feet up the kitchen chimney. The draught from the chimney so far ventilated the flue that the smell from the bedroom, almost immediately after this arrangement was completed, ceased to exist. This scheme might materially help the solving of the liquid sewage problem, inasmuch as less water would be required to be passed along the sewers, and the slightly diluted sewage would, in its passage along, meet and dissolve from the coal smoke the ammoniacal salts which it contains, and the liquid might afterwards be treated directly for the recovery of these valuable products, which might be converted into the products richer in plant nourishment than the richest guanos.

Mr. Spence calculated that, by the burning of two millions of tons of coal in Manchester and Salford, about 20,000 tons of sulphate of ammonia could be produced, all of which, by his scheme, would be deposited in the tunnels and added to the sewage; and taking its average value at £15 per ton, would be worth £300,000, a heavy item to be thrown away uselessly and harmfully into the atmosphere, and, taking this in combination with the manurial value of the sewage, he considers that about £800,000 per annum might be realized by its sale. This, then, is another point which adds to the prospect of the financial success of such a scheme.

Other schemes have been suggested for the prevention of the pollution of our atmosphere, such as the preparation of purified coal gas from the coal, and using that as fuel. We are, however, a conservative people, and any material improve-

ments which are made must be done slowly and cautiously. I understand that Mr. Spence's system has been adopted at the Assize Courts in Manchester, where all the fires are ventilated into the same chimney, and that it will be adopted in the New Law Courts of London; whilst I believe that the full scheme advocated by Mr. Spence and others will ultimately be adopted in all large towns. Mr. Spence, who has spent much time during his life in the study of those matters, and who has a thorough knowledge of what work can be done by large chimneys, &c., informed me a few days ago, that he saw no reason, since he first brought forward that scheme, twenty-two years ago, of altering his opinion as to its practicability. The point, however, which I wish to be most clearly brought forward in this paper, is that, considering the vast and growing importance of the subject, some means should at once be taken to mitigate the nuisance, and reduce the impurities which we are now compelled to breathe in our large towns, and this may be done with certainty, simplicity and little expense, by joining all the chimneys in each block or line of buildings together, for instance, and connecting them into one chimney, which should stand considerably above the tops of the houses. Much of the impurities would be condensed and deposited in the flues before the smoke reached the chimney, and it would allow of greater diffusion of the noxious gases which escaped from its increased height, so that we should require to inhale less of them, and further it would increase the draught in every chimney so joined. The draught could then be easily regulated by dampers; and, lastly, it would be preparing the way to centralize the points at which the smoke should be allowed to escape, until lastly a chimney of immense height would take the smoke and fumes sufficiently high to prevent their ever reaching our level.

Weighed against the comparatively small expense which the carrying out of some such plan would entail, the immense benefit which would be derived from it, it certainly seems astonishing that something has not before been attempted in this direction. Each fraction of a percentage of impurity which we are compelled to breathe adds considerably more to the drag which is thus put on the wheels of human energy, and it kills the plants and trees which were provided by nature to purify the air from the necessary contamination produced from the breathing of animals, and to refresh and delight the eye and the mind by their beauty. The smoke also prevents the free entrance to our towns of the rays of the health-giving sun.

The benefits which might be derived from having a pure atmosphere in our large towns can scarcely be estimated, and I trust that this Society will spare some of its talent and energy in the future in trying to improve the sanitary condition of the air of our large towns.

RIVER CONTROL AND MANAGEMENT.*

By J. CLARKE HAWKSHAW.

From "Engineering."

The question of the management of the rivers of this country is one that is yearly assuming more importance. Royal Commissions and Committees of both Houses of Parliament have been appointed of late years to consider the best means of preventing pollution, of preventing floods, and of providing for the control and management of rivers. Many facts have been accumulated and some valuable recommendations have been made which would go far to remedy the existing evils. It is to be hoped that decisive legislation will not be long deferred.

During the last three years I have had occasion to examine the greater part of three river basins, those of the Clyde, the Witham, and the Upper Thames, for the purpose of obtaining detailed in-

* Read before Section G of the British Association: Dublin meeting.

formation for my father who has reported on them. In the case of the Cylde he was appointed Royal Commissioner to inquire into the best means of preventing the pollution of that river, in the other cases he was called upon to report on the best means of preventing floods. This being the case (and seeing that the question of water supply has formed the leading topic of the president's address in this section) I have thought that a few remarks on river control may prove of some interest to members of the section. In this country, although so many great engineering works have been carried out, we have little cause to congratulate ourselves on the condition of our rivers. Periodically thousands of acres of land are flooded, and crops on them are destroyed and villages and towns are inundated. In January, 1877, 40,000 acres of land above Lincoln and between Lincoln and Boston were flooded by the River Witham. Two months later I saw the water still standing on parts of the fens, and the people only then returning to their homes from which they had been driven at a moment's notice. In some parts the water had been from 5 feet to 6 feet deep. By the same flood nearly 100 acres of the city of Lincoln were flooded to a depth in places of 3 feet. Twice in the previous year the River Trent flooded the town of Burton, rendering many of the principal streets impassable owing to the strong current running 3 feet deep through them. The banks of the Thames from near its source as far as London, and including parts of London itself, are liable to be flooded. These cases have come under my own notice, but each river valley could supply similar or even more striking ones. Yet I venture to think that engineers are not to blame for this state of things. If they cannot cope with the rivers of this country what hope is there that the great rivers of India or of other parts of the world will be controlled. The Mississippi drains an area of 1,147,000 square miles, which is more than 180 times as large as the area drained by the Thames, and discharges on an average 800,000 cubic feet per second during the flood period which lasts six months. A maximum flood discharge of the Thames might reach a fortieth part of the above amount for a few hours in the course of twenty years. Yet American enterprise has not shrunk from attempting to control the waters of the Mississippi; embankments were made at New Orleans 150 years ago, but the Americans are now wisely beginning to deal with the mouth of the river, a course which has been too often neglected in this country. By making jetties, one of which is now more than two miles long, formed of piles and matrasses of fascine work, they are constraining the waters of one of the main outlets of the river to flow in a deep straight course to the sea. Already the depth of water on the bar has been increased from 8 feet to 19 feet.

I will not further describe this great work, though I am much tempted to do so. Its progress should be watched by all those interested in river engineering. Although much still remains to be done before even the great rivers of Europe will be brought under control, much progress has been made where local interests have been made subservient to those of the community in general. The fault in this country most frequently lies in the existence of numerous authorities each with limited jurisdiction over the same river. Rivers can rarely be dealt with to any good purpose piecemeal, but should more often be dealt with as a whole from their source to the sea. And, moreover, whatever is done to them should be done with a view to making them as efficient as possible for the three purposes which they should serve, viz., for drainage, for navigation, and for water supply. The first of these three is the most important, for on it the existence of a river depends. It need not necessarily serve the purpose of navigation; water may be obtained for the population within its basin from other sources, such as wells, or from districts without its basin, but a river cannot escape, being the natural and proper outlet for the land drainage of its basin. I shall attempt to discuss these three functions of a river separately, though I believe they cannot fitly be separated, whether we regard the works which are proper to them or the organization which will best lead to their being well performed. I have said that I do not think engineers are to blame for the condition of our rivers. This is certainly

not the case in respect of floods. A flood occurs and an engineer is called in by a local board or committee of landowners, and is asked to report and suggest a remedy for the evil. He has generally to consider but a small portion of the river, and that only in so far as it affects the interests of his employers, who have possibly no jurisdiction or at most only a partial jurisdiction over it. Proper surveys are not to be had, nor observations on the flow of water in the river, and they often have to be dispensed with in a great measure owing to the time it would take to make them and the cost. He is limited as to expenditure, yet a partial scheme of improvement must generally be costly. It can hardly fail to benefit some people higher up the river who are not called on to pay. Very often it will injure others lower down. While the report is being prepared, the ardor to remedy the evil cools. Some are not much injured by the flood, some not at all. The prospect of having to pay at once seems more distasteful than to risk the chance of having to suffer the same loss and discomfort at some uncertain future time. So the report is received and is discussed. The cost of the proposed works is too great, and there the matter ends till another bad flood comes and the whole process is gone through again.

Engineers have made more reports on drainage than on almost any other subject. These reports lead to much unprofitable expenditure when special observations and special surveys have to be made for them. Such work can easily be so thoroughly done as to be available a second time. For a succeeding report it often has to be done over again.

The Ordnance survey maps on the 6-inch scale supply much of the information that is wanted, but for many districts no maps have been published on the 6-inch scale, and for some we have nothing but the 1-inch scale maps 30 or 40 years old.

Much of the district drained by the Witham is not correctly shown on any map. The Ordnance survey of it was made in 1824, and the appearance of the fen land has totally changed since then. When the Thames Valley Drainage Commissioners were appointed in 1871, by Act of Parliament, to deal with the rivers and flooded parts of the Upper Thames Valley, they could not properly perform the duties imposed upon them without a correct map of the district. The Ordnance survey of the district was made in 1828, on the scale of one inch to a mile. The commissioners were forced to have a new survey made which took some years to make. The work was done by the Ordnance Survey Department, to whom the Commissioners paid several thousand pounds for it, so that in this case a special tax was laid on the landowners of the district under the commissioners for the benefit of the Ordnance Survey Department. This is one of many cases in which the want of system in the management of our rivers has led to a limited district having to bear burdens which should rightly have been borne by larger districts, or, in this case, by the whole community.

In England, alone, the extent of land which directly depends for its drainage on the state in which the main river channels are maintained is very great. The fens of the Eastern Counties extend over 830,000 acres, and this forms but a portion of the land throughout the country, some of which would be relieved from disastrous floods, and all of which would be better drained if the river channels were dealt with on some systematic plan, and were not, as at present, either entirely neglected or dealt with piecemeal.

Much of the fen land has no natural drainage, but no less does the cost of draining it by mechanical or other means depend on the state in which the river channels are maintained. To drain the fen lands in the first instance it was necessary that considerable areas should be dealt with at once, and this led to the formation of numberless drainage districts. Each of these provides for its own drainage as best it can with regard to its own interests only, and often to the detriment of adjoining interests. Occasionally several of them may unite for some special purpose, but they rarely co-operate heartily. They have not always control over the main channel into which they drain, so that they are liable to be flooded from causes beyond their own control. Before the advent of railways river channels were valuable as means of inland transport, and were

maintained for that purpose. But traffic on rivers has fallen off, and the old conservancy boards have not funds to maintain the channels, though they still have jurisdiction over them, and drainage suffers in consequence.

The River Witham may be taken as a good example to illustrate the variety of jurisdictions to which some of our rivers are subject. The Witham is between 60 miles and 70 miles long, and drains an area of 1079 square miles. It flows past the towns of Grantham, Lincoln, and Boston, entering the Wash about seven miles below Lincoln. For the last 40 miles below Lincoln it is embanked on both sides to prevent its flooding the low land adjoining. Parts of its tributaries, both above and below Lincoln, are likewise embanked for the same purpose. For some miles below Grantham the Witham is under the Grantham Sewers Board, from thence to Lincoln it is under the Lincoln Sewers Board. These boards have rating powers over the lands adjoining the river; the Lincoln authority rates all lands below flood level and less than 2 feet above that level. The part of the basin of the Witham which lies to the north-west of Lincoln is drained by a tributary the Till, which does not, however, discharge directly into the Witham but into the Fossdyke, a navigation connecting the Witham at Lincoln with the Trent at Torksey. The Till is under the Lincoln Sewers Board, but the Fossdyke is under the Great Northern Railway Company. The lands adjoining the Fossdyke, which are below the level of its waters, are divided into two districts, the north and south-west districts, under separate Boards. The only outlet for the drainage of these districts, 7,000 acres in extent, is through a small culvert under the Witham at Lincoln, which is not large enough in time of floods. Besides this, these districts are liable to be overflowed by the waters of the Till, the Fossdyke, and the Witham by the failure of banks over which they have no control. Between Lincoln and Boston, the Witham is a navigation subject to the Great Northern Railway, and neither the Lincoln Sewers Board nor the West District Drainages have any control over it, but the jurisdiction of the Lincoln Sewers Board extends over the Lanworth river, the principal tributary of the Witham between Lincoln and Boston, and likewise over a cut called the Sincil Dyke, which passes through Lincoln and connects the Witham with the South Delph, which is under the Great Northern Railway, and which enters the Witham about seven miles below Lincoln. The Lincoln Sewers Board are prohibited by Act of Parliament from making a proper outlet for the Witham waters into the Sincil Dyke, and as their only other outlet, the navigable channel of the Witham, is not under their jurisdiction and is insufficient in time of floods, this arrangement insures a large tract of land above Lincoln, and part of the town itself being flooded at times. From Lincoln to Boston the Witham is under the Great Northern Railway for navigation purposes, but it is under another authority called the Witham Commissioners for drainage purposes. If the Witham banks were to fail at any point as they have done the adjoining lands are flooded. These banks, more than 60 miles in length, are maintained partly by the Great Northern Railway, partly by the Witham Commissioners, and partly by the adjoining landowners. From Boston to the sea the Witham is partly under the jurisdiction of the Boston Harbor Commissioners and partly under the Witham Commissioners. Below Lincoln the lands adjoining the Witham are divided for drainage purposes into seven large districts which are again divided into many smaller ones, each of which has its own body of commissioners and other officers, and power to rate for drainage purposes. Lastly, there are two affluents of the Witham below Lincoln, which have been made into navigations under separate authorities, the Horncastle navigation and Heaford Canal.

Thus there are no less than seventeen sets of Commissioners or other authorities having jurisdiction over the Witham, and for drainage purposes over the lands adjoining it between Grantham and the sea, exclusive of the commissioners of the smaller drainage districts. If we included them the above number would be doubled. In the first large district below Lincoln there are eight sets of commissioners. The ratepayers and others whom they represent all suffer periodically, some to a disastrous extent, from

the present state of the river. Yet these heterogeneous bodies never have united for the purpose of making the main channels, in which they are all interested, contribute to their common welfare, instead of being, as now, a constant source of danger and loss.

It is only by a long series of observations made at different points along a river channel that we can ascertain the maximum quantities of water which the different parts of the channel may have to convey. The quantity varies from day to day and from year to year, being, in the first instance, dependent on the rainfall. The rate of flow is liable to be altered by the operations of man in tilling the land. The cutting of open drains and ditches and subsoil drainage for agricultural purposes, as well as the destruction of waste and woodland, all tend to alter the rate at which the water falling on the land is conveyed to the sea. The observations required to ascertain the rate of flow are not likely to be made by local bodies elected for drainage purposes with partial jurisdiction, local interests, and limited funds. Until our rivers are placed under bodies who have wider interest and larger funds, and who will make systematic observations, we shall not know as nearly as we ought the quantity of water which the different parts of the river channels should be made to convey, and yet this is among one of the first things which we ought to know. I only know of one extensive set of observations on the flow of water in the River Thames, and they were made for the purposes of water supply, and at one place only.

The channel of a river, whether it be a channel excavated beneath the natural surface of the land, or a channel partly so formed and partly by banks rising above the surface of the land, should be large enough to convey all the water which systematic observation has shown it may have to convey. Some riparian owners are of opinion that floods should not be prevented or only partially prevented, and as long as any part of our rivers are under the control of those who hold such opinions, their channels will not be made as perfect as they should be for drainage purposes. It is generally held that grass lands are benefited by being laid under water at times, but there must be a wrong time and a right time for so treating them. As long as it is left to a chance flood to flow over the meadows, there will be a possibility of its happening at a wrong time. Given the water it will rarely be beyond the power of the engineer to distribute it by gravitation when it is wanted over any of the lands which are now ready to trust to floods for their supply.

In considering the works necessary for the prevention of floods, the question arises whether the extra sectional area required in a river channel should be obtained wholly by increasing the section of the channel below the surface of the land, or partly by that means and partly by embankments. When the fall is small, the latter plan must often be resorted to, and also when the flow in flood time is very large, as compared with the ordinary flow. As there is often some saving in making use of embankments, there will be a tendency to adopt them, more especially among local drainage authorities having small means. But unless they are well maintained, the first saving is soon swept away by the damage done by floods to which their neglect gives rise. As a rule, the larger the jurisdiction of the authorities having to maintain the banks, the better are they maintained. This is noticeable on the Witham, where the worst banks are those kept up by small local drainages.

Although it is safer to depend on a channel excavated below the surface of the land which lowers the flood line, embankments might be economically used, and without the danger which so often now attends them, if rivers were under wider jurisdictions. Formerly river channels throughout a great part of their length were maintained under navigation authorities by the tolls derived from the traffic on them. As the inland navigation tolls have dwindled away since the construction of railways, so the channels inland are no longer maintained. But although navigation authorities now may pay little attention to their inland waterways, large sums are still spent in many cases in maintaining the lower or tidal portions of rivers, and while they do this only with a view of navigation requirements, they at the same time render the channels efficient

for drainage purposes. It is fortunate indeed for those interested in land drainage that this is so, for they have often no jurisdiction over the tidal parts of their rivers.

Some districts, however, are not so fortunate. On the River Witham not only has the inland navigation fallen off, but Boston has ceased to be one of the chief ports of the country. The river channel through and below Boston is now often choked up with silt and mud, which increases the difficulties and cost of draining the lands above. There is sometimes 12 feet of silt against the seaside of the doors of the Grand Sluice at Boston at the end of summer, so that the outlet to the Witham is completely blocked up. When the autumn rains come suddenly, a flood is almost sure to occur between Lincoln and Boston, owing to this state of things. If Boston were a thriving port, the river would never be allowed to remain in the state in which it is. The drainage authorities have only a partial jurisdiction over this part of the river, and the funds of the harbor authorities are so limited that between the two little is done.

At the mouth of a river the requirements of drainage and navigation are much the same. For drainage purposes the low water level in the river from the sea upwards should be kept as low as possible. Generally this end will be attained by straightening the river's course or by enlarging and regulating the channel. But this will at the same time increase the rate of tidal propagation and prolong its duration, which will benefit navigation. The Eau brink cut on the lower part of the Ouse was made for drainage purposes. It lowered the low-water level at Denver Sluice 7 feet, which was an immense gain for drainage, but it was also a great benefit to the navigation. Many cases could be given where the low-water level has been lowered far up a river as on the Tay, the Ribble, the Lune, by works carried out for navigation purposes, and I think it will generally be found that the works required for navigation may be so designed as to improve a river at the same time for drainage purposes.

Moreover, although it has been found unprofitable to maintain many rivers in a condition suitable for navigation, if rivers were once again made to fulfill efficiently the agricultural requirements of the adjoining lands, they could, with very little additional expenditure in many cases, be made available for navigation, so much is there in common between the works required for the two purposes. The difficulty of finding a good and sufficient supply of water for centers of population throughout the country is constantly increasing. Water from rivers, except when obtained from near their source, can rarely be used, owing to the filthy condition to which they have been reduced by the increase of population and extension of manufactures uncontroled by legislation. In some districts, as I found to be the case in the Clyde valley, manufactories have been established high up the purest streams, which are now polluted almost from their source.

Much improvement cannot be looked for as long as it is left to voluntary action to effect it. There is little encouragement to try and prevent the pollution of water which flows past your door if you are powerless to prevent its being polluted to a greater extent before it comes there.

One of the principal difficulties which stand in the way of legislation is the absence of a body to enforce any enactments for the prevention of pollution of rivers. If, however, a body of Commissioners were appointed for each river basin, this duty is one which might be imposed on them in connection with their other duties.

In the case of the River Thames some stringent measures were rendered necessary, owing to the difficulty of finding a source of water supply other than the Thames for so large a population as that of London. Advantage was taken of the existence of the Thames Conservancy, a navigation authority having jurisdiction over the greater part of the Thames but none over its tributaries. Their jurisdiction for the prevention of pollution was at first, confined to the part of the Thames for which they were the navigation authority, and to the tributaries of that part for a distance of three miles from the river. This distance was afterwards extended to five miles, and this year to ten miles from the river. The Thames Conservancy at once prohibited certain

towns on the banks of the Thames from discharging their sewage into it, and they have taken other steps to improve the quality of the water. This is good so far as it goes; without it the Thames would soon become unfit to supply London with drinking water. But there is still a large population living on the banks of its tributaries of the Thames which is not prohibited from discharging sewage into them, but is permitted to pollute the water which is drunk by three millions of people in London.

In order to provide the Thames Conservancy with funds to be used by them in carrying out the new duties imposed upon them of preventing pollution, London has been taxed through the water-works companies, who yearly pay a large sum to the Conservancy, so that London now bears the whole burden of enforcing the partial purification of the Thames.

The purity of river water is directly affected by its depth. Weeds, which by their decay pollute the water, do not greatly flourish in depths over five feet. Navigation is rarely compatible with a depth of much less than five feet, so that any works which may be necessary, as is generally the case inland, to maintain that depth or a greater one for navigation purposes, will tend to benefit the water supply; the cleansing of channels from mud and weeds is also necessary for land drainage purposes. Each water-course if neglected may become a source of pollution, so that well-regulated land drainage works tend to some extent to improve the quality of the water.

Quality is not all that is required in the water of our rivers regarded as sources of water supply. Quantity is also necessary. Weirs for providing the depth of water required for navigation tend to maintain a large volume of water in the river in dry seasons. And generally it will be found, I think, that the works required to make a river serve satisfactorily the purposes of land drainage and navigation will tend to improve the quality of the water, and in many cases increase its available quantity by increasing the storage capacity of the river.

I have endeavored so far, in a somewhat fragmentary manner I am afraid, to show the connection there is between the works required to make a river fulfill the purposes of drainage, navigation and water supply, and some of the evils which arise when such works are carried out, as is now the case, by numerous separate authorities having only limited jurisdiction. I will now state what appears to me to be some of the requirements for improving the existing state of things. Generally I think that Parliament should make it compulsory for each river basin to appoint a body of commissioners who should have jurisdiction over a river from its source to the sea, and over all its tributaries and sources of water supply. Their powers should be exercised for the purpose of drainage, navigation, and for the preservation of the water and maintenance of its purity for the purposes of water supply. These commissioners could divide the drainage area into districts, so that local matters might be referred to local commissioners, but it is essential that all works should be carried out under the general supervision of the central authority.

The question remains as to how the money should be raised to carry out the works to be done by the commissioners for each river basin. When large or thickly populated districts have to be dealt with, any rate or tax must press more on some than on others. But it ought to be possible to find some means of adjusting the cost of maintaining river channels in an efficient state that would be fairer to all than the present method.

Every acre of land in a river basin either directly contributes its supply of water to the river channels or in some way affects the supply from adjoining acres. As we cannot ascertain the effect of each acre on the supply of water, it appears to me it would be most just to tax each acre according to its value, that is, to levy a tax on all the land within the river basin in proportion to its annual value for whatever purpose it is used, and whether it be covered by buildings or not.

But as some lands are now subject to floods and would be at once improved in value by their prevention, such lands should, in the first case, be taxed at a higher rate, but that when the works necessary to prevent such damage by floods were once done and paid for, the general fund for the maintenance of the

river channels should be raised by an equal rate levied on all land according to its annual value.

A small rate levied over all the acreage of a river basin would produce a large sum. Taking the area of the Thames basin at 6160 square miles, or close upon four million acres, and assuming the average annual value to be £2 per acre, which is probably too low, a rate of one penny in the pound would produce £33,000 a year.

When a river was once made efficient the navigation as far as purely navigation works are concerned would probably more than maintain itself, as the river channels would have to be maintained for drainage works alone. Any surplus might go to the general fund. As regards the use of the water the river channels should be free to all for the purposes of recreation, and as all districts would pay to maintain the quantity and purity of the water, all should have a right to use the water subject to not prejudicing others. Lastly, as regards pollution, no claim for compensation for desisting from it should be admitted. The principle of dealing summarily with offenders has already been adopted on the Thames and elsewhere. Those still offending should be similarly dealt with.

DRAINING AND IMPROVING 1,500,000 ACRES OF DESERT LAND.

By GEORGE WILSON, M. Inst. C. E.

From Proceedings of the Institution of Civil Engineers.

In the west of France, close to the important town of Bordeaux, there existed, previously to the year 1850, an immense territory of uninhabited desert lands, comprising nearly 2,000,000 acres superficial area, and designated "Les Landes de Gascogne." The territory is triangular in shape. The base, commencing at a point close to the mouth of the river La Gironde, extends to the south of that river along the shores of the Bay of Biscay for a distance of about 140 miles; while one of the sides, passing by and about 5 miles distant from Bordeaux, is approximately parallel to the course of the river La Gironde for a distance of about 120 miles from its mouth.

The desert lands of this territory resembled a vast and nearly horizontal plain at an average altitude of about 330 feet above the level of the sea. The surface of the ground consists of poor sandy soil, without any trace of clay or calcareous matter in its composition, varying in depth from 12 to 20 inches. Under the surface soil there is an impermeable layer of silicious sand, agglomerated by vegetable matter, varying in thickness from 16 to 20 inches, forming a sort of organic cement, which is known in the country under the name of "alios." The substratum under this layer of "alios" consists of compact white sand impregnated with water. There does not exist on the surface of the plain any spring, nor is there any trace of water during the summer. During the winter, however, previously to the execution of the drainage works, the abundant rain of that coast, experienced for more than six months, falling on the plain, could neither flow off its surface, nor percolate sufficiently through crevices of the impermeable layer of alios into the substrata. The water remained stagnant until it was evaporated by the heat of summer. Thus the constant inundation during the winter, and the dryness of the hot sandy soil during the summer, rendered the ground sterile for any kind of cultivation, and extremely unhealthy for the inhabitants and for animals to exist or work on it.

Previous to 1850, several attempts had been made, and much money expended, in endeavoring to improve and drain portions of this desert, all of which, however, resulted in failures, and caused the impression to prevail, that the soil was of little value, certainly not worthy the expenditure necessary to drain and improve it for the purpose of cultivation. In

1849 a report appeared by M. Chambrelent, Ingénieur en Chef des Ponts et Chaussées, describing the results of complete studies, surveys, levels, borings, &c., of the desert, which he was officially commissioned to carry out, extending over seven years. From these studies the exact physical features of the desert were developed, and at once indicated the mode of improving and rendering it profitable for cultivation. It was shown that, upon the whole extent of the apparently horizontal plain, there exists from the summit towards the valleys extremely regular slopes, but so small that the least irregularities of the ground hinder the water from following the normal declivities.

The average inclination of the ground is about 1 in 1,000, but in some parts it does not exceeed 1 in 2,000, and continues always in the same directions towards the valleys without any noticeable change or break. The small irregularities on the surface, which hinder the flow of the water, are never more than 12 to 16 inches above or below the normal slopes of the plain. Therefore a drain 20 to 24 inches deep, with its bottom parallel with the general slope of the ground, can be made along its entire required length without necessitating a cutting of more than from 24 to 28 inches, in order to allow the water from rainfall to pass perfectly away. Moreover, a ditch extending along such sandy permeable ground will drain the surface to a considerable distance on each side of it, and as its average inclination will be about 1 in 1,000, the water will percolate slowly and regularly to it without causing injury to the sides. In consequence of the permeability of the surface soil very few drains are required.

It had been noticed that there existed in a few isolated places of the desert clusters of fir trees, locally termed "Pignades," which flourished remarkably well; also that the ground at such places was slightly higher than the normal inclination of the plain, and was therefore drained naturally, and that both the surface soil and the subsoil where the trees flourished were physically and chemically, as shown by analyses, similar to the ground of the entire plain. It was thus evident that the desert only required to be properly drained in order to cultivate it for a forest, and for it to produce trees similar to the clusters or "Pignades" actually growing. There were, however, at the time, numerous objections against these deductions as to the practicability of the drainage and improvement scheme. It was contended that drains cut through such sandy ground would soon be silted up, from the effects of the wind on the sandy plain, and be trodden in by cattle, or destroyed by the drainage water percolating through their sides. A further objection was urged, that the drains would have to be made of such great lengths that the water could not be caused to flow along them to the distant valleys.

To meet these objections, and to test the practicability of the scheme, it was decided to make a trial. A superficial area was selected of about 1,250 acres of the desert, nearly on the summit, and in the middle of one of the most sterile parts, being in every sense under the most unfavorable conditions for draining the soil. At the time of commencing operations the ground was covered with stagnant water, so that it was necessary to move about on the stilts used in the country. The effects of the drains was immediately satisfactory. The ground was so well drained that during the heaviest winter rain no stagnant water existed on it, and the water flowed away abundantly and with great regularity. All the rain water percolated immediately through the soil into the drains, and none was noticed to flow along the surface. At present, although it is more than twenty-seven years since these drains were executed, yet such is the slowness and regularity with which the water percolates into and along them, that they have not silted up nor have their sides been injured.

Shortly after this trial ground was drained, it was sown with fir seeds and acorns, and the results were so satisfactory that in the following year numerous proprietors of portions of the desert carried out similar works, and obtained an equal success; and in less than five years an area of more than 50,000 acres was drained and cultivated for forests. As regards the first sowing on the trial ground, in five years fir and oak trees had grown over the whole area to a height of nearly 12 feet, and each tree

was about 12 inches in circumference above the ground.

After proving the practicability of draining the territory, the important question arose as to the possibility of obtaining wholesome drinking-water for the inhabitants. There are no springs of water, and the potable water hitherto obtained was derived from wells dug through the "alios" to a depth of about 4 feet below the surface of the ground. The bad quality of the water caused even worse fevers and other diseases than the unhealthiness arising from the marshy nature of the soil. Borings were made to various depths below the surface through the "alios" into the substratum of compact white sand, and on analyzing the water, it was shown that the water increased in purity with the depth. At a depth of about thirteen feet from the surface the water was practically fit for drinking purposes provided it was caused to filter at the bottom of the well, by ascending through an artificial bed of broken calcareous small stones and clay gravel. Wells have therefore been sunk, wherever required over the territory, to a depth of about thirteen feet, and the sides built with masonry laid in cement so as to be perfectly and permanently watertight. The water can thus only ascend the bottom of each well after filtering through an artificial layer of broken stones (generally the *débris* from dressing the stones for lining the well) and clay gravel. The organic matter found in the water of the old wells was as much as $\frac{1}{2917}$ part by weight, or about 2.4 grains per gallon, while the water drawn from the new wells only contained organic matter equal to $\frac{1}{7000}$ the weight of the water, or 0.14 grain per gallon.

Extensive portions of the deserts belonged to one hundred and sixty-two communes, and were so intermingled with the property of private owners as to be a hinderance to the latter carrying out their drainage works effectually until such portions were drained. To facilitate and ensure the communal possessions being drained, and to construct the necessary roads of communication over such a large territory, the Government, in 1857, made the following special laws: "The waste lands of the communes of the two departments, La Gironde and Les Landes, shall be drained and improved at the expense of the communes. In case of inability, or refusal on the part of the communes, to carry out these works, the same shall be done at the expense of the State, to be repaid the advance made in principal and interest out of the produce and cultivation." The law also provided for the construction of the roads of communication by the State.

So soon as the above law was promulgated, a general plan was prepared for draining the waste lands belonging to the communes, comprising about 720,000 acres, irrespective of about 864,000 acres belonging to private individuals, which remained to be drained. The plan comprised the execution of the principal canals necessary for receiving and carrying away the water flowing from the land drains. The fixing of these canals over ground almost flat presented many difficulties. The whole country had to be again carefully leveled, which showed that the inclination of some portions of the plain was not more than one in 2,000. The inclinations of the canals had to be calculated in such a manner so as to be sufficient for the proper drainage of the water resulting from a varying rainfall, and yet not such as would cause a current injurious to the canals constructed through ground of such a sandy nature. The main canals in the department of La Gironde were 13 feet to 16.4 feet wide at the bottom, and their total lengths amounted to 1,365 miles. The canal between the pools or lakes Lacanau and Langourarde, bordering the sand dunes on the coast (which had to be constructed for a distance of $\frac{5}{8}$ mile through a marsh always covered with water), is 39.4 feet wide at the bottom, 6.3 miles in length, and has an inclination of 1 in 4,000; while the canal made between the pools or lakes d'Hourtins and Lacanau, through the middle of a marsh for a distance of 5.2 miles, is 23 feet wide at the bottom. The two last-mentioned canals receive the drainage water of 200,000 acres superficial area.

In 1865 the drainage works of the communal lands were completed at a cost of about 1*s.* 10*d.* per acre drained. Out of the total area of communal lands comprising 720,000 acres, 466,000 acres have been sold from time to time to meet expenses, so that the funds of the State

were not drawn upon. Some portions, previous to being drained, were disposed of for a sum of for a sum of £537,278, which sum was applied for the following purposes:

	£
Draining	35,739
Sowing land with seed	27,272
Construction of new and restoration of old churches	95,660
Construction of new and restoration of old parsonages	27,082
Construction of new and restoration of old town-halls and school-houses	65,455
Subscriptions, subventions and allowances for the construction of roads	79,489
Various expenses—construction of wells, removing cemeteries from market towns and villages	32,471
Communal funds invested in rentes of the State	174,110
Total	537,278

With the exception of lands situated in various parts of the plain, amounting to about 70,876 acres, respecting which there are, amongst other causes, disputes as to the boundaries, the whole of this formerly desert territory is now under forest cultivation, consisting mainly of fir trees. Oak trees, although growing equally well, and more profitable in the course of time, are nevertheless more expensive to cultivate during the first few years, and consequently the ground has only been to a limited extent used for their cultivation. Everywhere and in every respect the results of forest cultivation are highly satisfactory, showing the most vigorous vegetation and growth, and developing equally well each successive year. The only cause for anxiety arises from accidents and losses by fire. From 1865 to 1870 serious losses occurred from that cause, destroying 25,000 acres of forest. Since 1870 such fires have somewhat diminished, and the Government is adopting special measures to prevent these disasters. It might be considered that the production from forests, comprising an area of 1,500,000 acres, would so much diminish the price of timber, that profitable markets could not be found. Time and experience have, however, proved that no fears need be entertained on this question. Each year the purposes and the appliances for the production of such forests increase, and the selling prices augment accordingly. The timber of these forests is not only sold in France, but is exported to England and Scotland, and even to America and Spain. In England large quantities are sold for props, &c., used in coalpits, in pieces each about 6½ feet long and 3½ inches in diameter. The younger trees and branches are sold for the fuel throughout France. Large quantities are used for making wine cases and barrels. A mill has recently been successfully set to work, making 1,000 tons of pulp per annum, for the manufacture of paper direct from the fir tree, which industry it is anticipated will rapidly increase, inasmuch as the pulp produced is of brilliant whiteness. Another industry is being developed in manufacturing, by the aid of recent discoveries, rectified essence of turpentine from fir trees, for the purpose of illumination, the light produced being considered superior to, and more economical than that, from any of the oils or from petroleum. The above are some of the objects for which the production of the forests are used up to the first twenty-five years of their growth, after which the timber is sufficiently large to be applied to more extended purposes, such as general carpenters' work, sleepers for railways, telegraph posts, &c. For the latter purpose large quantities are already being used in France, and exported to Algeria and Spain. In 1877 the timber on a plot of ground of 11 acres superficial area, which was in its twenty-eighth year of growth, was sold for £192, equal to £17 9*s.* per acre; but this plot of timber was favorably situated for being carried away.

At an average age of thirty years the present forests, comprising 1,514,110 acres, are estimated to produce timber worth at a minimum price £9 15*s.*, and in seventy years £32 10*s.* per acre, respectively equivalent to a total value of about 14¾ and 49¼ millions of pounds sterling for the produce of those lands which were recently a desert. This valuation is based on the probable casualties by fire, and would consequently be considerably higher if the dread of losses from that cause did not exist.

Independently of these satisfactory results, the drainage of the desert, and the construction of d eper wells for the supply of wholesome drinking water, have changed the district into one of the healthiest in France; whereas previously, the

inhabitants suffered constantly, from fevers and other fatal diseases, causing the death-rate to be exceedingly high.

The maintenance of the main canals, of a total length of 1,365 miles, is under the inspection of the administrations of the two departments, La Gironde and Les Landes, who give orders to the mayors of the communes and proprietors of lands bordering the canals to clean them out when necessary, or to remove impediments in them from whatever cause, inasmuch as it is of importance that canals of such great lengths, small inclinations, and dug in sandy soil, should always be maintained in good order.

ELEMENTS OF THE MATHEMATICAL THEORY OF FLUID MOTION.

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The next case of vortex motion that we shall consider is that in which the vortex lines are circles having their centers in the axis of z. The direction of the axis of rotation of each fluid particle will lie in a plane at right angles to z and be parallel to the tangent to the vortex line at that point. The reader will do well to observe the motion of a ring of tobacco smoke; he will see that the ring seems to be turning inside out, each particle moving in a plane passing through the axis of the ring and revolving in a circle whose axis is in the direction of the tangent to the ring. He will also observe that there is no motion around the axis of the ring. Now if we introduce polar co-ordinates ρ and ϑ we have, evidently,

$$x=\rho\cos\vartheta \quad y=\rho\sin\vartheta$$

and for the rotations we may obviously write

$$\xi=-\lambda\sin\vartheta,\ \eta=\lambda\cos\vartheta,\ \zeta=o,$$

when λ is not a function of ϑ. The equation of the path of the particle is evidently one between x, y, z where x and y are connected by the relation

$$\rho=\sqrt{x^2+y^2}$$

so that the equation of the path can be made to depend only upon ρ and z. Resuming now our equations

$$u=\frac{d\mathrm{W}}{dy}-\frac{d\mathrm{V}}{dz},\ v=\frac{d\mathrm{U}}{dz}-\frac{d\mathrm{W}}{dx},\ w=\frac{d\mathrm{V}}{dx}-\frac{d\mathrm{U}}{dy}$$

and observing that

$$\mathrm{W}=\frac{1}{2\pi}\int\frac{\zeta d\tau}{r}$$

these give

$$\mathrm{W}=o,\ u=-\frac{d\mathrm{V}}{dz},\ v=\frac{d\mathrm{U}}{dz}\qquad w=\frac{d\mathrm{V}}{dx}-\frac{d\mathrm{U}}{dy}$$

If now we assume an element $d\tau'$ given by the co-ordinates ϑ', ρ', z' and for which λ becomes λ'; also denote by r its distance from the point (ρ, ϑ, z) or $(x\,y\,z)$. Now our equations for U and V are

$$\mathrm{U}=\frac{1}{2u}\int\frac{\xi' d\tau'}{r}=-\frac{1}{2\pi}\int\frac{\lambda'\sin\vartheta' d\tau'}{r}$$

$$\mathrm{V}=\frac{1}{2\pi}\int\frac{\eta' d\tau'}{r}=\frac{1}{2\pi}\int\frac{\lambda'\cos\vartheta' d\tau'}{r}$$

When

$$d\tau'=dx'\,dy'\,dz'=\rho' d\rho'\,dz'\,d\vartheta'$$

and

$$r=\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos(\vartheta'-\vartheta)}$$

Now make for convenience

$$\vartheta'-\vartheta=\varphi$$

this gives $\vartheta'=\varphi+\vartheta$ and $d\vartheta'=d\varphi$ therefore we have again for U and V,

$$\mathrm{U}=-\frac{1}{2\pi}\int\rho' d\rho'\int dz'\int\frac{\lambda'[\sin\varphi\cos\vartheta+\cos\varphi\sin\vartheta]d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

$$\mathrm{V}=\frac{1}{2\pi}\int\rho' d\rho'\int dz'\int\frac{\lambda'[\sin\varphi\sin\vartheta-\cos\varphi\cos\vartheta]d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

In both cases we have the integral

$$\int \frac{\sin\varphi\, d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

this is to be taken between o and 2π, these being the limits of $\Im'-\Im$,

$$\int_0^{2\pi} \frac{\sin\varphi\, d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

$$=\log\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}\Big]_0^{2\pi}$$

$$=\frac{1}{2}\log\frac{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}=o$$

and our expressions are thus reduced to

$$U=-\frac{1}{2\pi}\int\rho'd\rho'\int dz'$$

$$\int\frac{\lambda'\cos\varphi\sin\Im\, d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

$$V=\frac{1}{2\pi}\int\rho'd\rho'\int dz'$$

$$\int\frac{\lambda'\cos\varphi\cos\Im\, d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

Denoting the common integral by Φ we have

$$U=-\frac{1}{2\pi}\int\rho'd\rho'\int dz'\lambda'\,\Phi\sin\Im$$

$$V=\frac{1}{2\pi}\int\rho'd\rho'\int dz'\lambda'\,\Phi\cos\Im$$

Φ will clearly be of the form

$$\Phi(z'-z,\rho,\rho').$$

U and V now differ only by a factor, in fact we have

$$V=-U\tan\Im.$$

So if we write

$$P=\frac{1}{2\pi}\int\int\rho'd\rho'dz'\lambda'\Phi$$

we will have briefly

$$U=-P\sin\Im,$$
$$V=\ P\cos\Im.$$

The value of the function Φ is not difficult to obtain; we have

$$\Phi=\int_0^{2\pi}\frac{\cos\varphi\, d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

We will find the integration much simplified by the introduction of a new variable Ψ defined by the equation

$$\Psi=\frac{\pi-\varphi}{2}$$

Then we have

$$\cos\varphi=-[1-2\sin^2\psi]$$
$$\cos\varphi d\varphi=2[1-2\sin^2\psi]d\psi$$

and for the limits we have when $\varphi=(o,2\pi),\psi=\left(\frac{\pi}{2},-\frac{\pi}{2}\right)$. Making this transformation we have

$$\Phi=2\int_{+\frac{\pi}{2}}^{-\frac{\pi}{2}}\frac{[1-2\sin^2\psi]d\psi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2+2\rho\rho'-4\rho\rho\sin^2\psi}}$$

or,

$$\Phi=-4\int_0^{\frac{\pi}{2}}\frac{[1-2\sin^2\Psi]d\Psi}{\sqrt{(z'-z)^2+(\rho+\rho')^2-4\rho\rho'\cos\varphi}}$$

$$=-4\int_0^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{(z'-z)^2+(\rho'+\rho)^2}\sqrt{1-\frac{4\rho\rho'\sin^2\Psi}{(z'-z)^2+(\rho'+\rho)^2}}}$$

$$+4\int_0^{\frac{\pi}{2}}\frac{2\sin^2\psi\, d\psi}{\sqrt{(z'-z)^2+(\rho'+\rho)^2}\sqrt{1-\frac{4\rho\rho'\sin^2\Psi}{(z'-z)^2+(\rho'+\rho)^2}}}$$

Make,

$$\frac{4\rho\rho'}{(z'-z)^2+(\rho'+\rho)^2}=k^2$$

then

$$\frac{1}{\sqrt{(z'-z)^2+(\rho'+\rho^2)}}=\frac{k}{2\sqrt{\rho\rho'}}$$

then

$$\Phi=-\frac{2k}{\sqrt{\rho\rho'}}\int_0^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{1-k^2\sin^2\psi}}$$

$$+\frac{2k}{\sqrt{\rho\rho'}}\int_0^{\frac{\pi}{2}}\frac{2\sin^2\psi\, d\psi}{\sqrt{1-k^2\sin^2\psi}}$$

The first of these is the complete elliptic integral of the first kind; we will, as

usual, denote it by K. Examine now the second integral; we have on multiplying it numerator and denominator by k

$$\frac{4}{k\sqrt{\rho\rho'}}\int_0^{\frac{\pi}{2}} \frac{k^2\sin^2\phi d\phi}{\sqrt{1-k^2\sin^2\Psi}}$$

$$= \frac{4}{k\sqrt{\rho\rho'}}\int_0^{\frac{\pi}{2}} \frac{1-(1-k^2\sin^2\phi)}{\sqrt{1-k^2\sin^2\phi}}d\psi$$

$$= \frac{4}{k\sqrt{\rho\rho'}}\int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{1-k^2\sin^2\phi}}$$

$$- \frac{4}{k\sqrt{\rho\rho'}}\int_0^{\frac{\pi}{2}} \sqrt{1-k^2\sin^2\phi}\,d\psi$$

The second of these is the elliptic integral of the second kind and denoted by E; we have then finally for Φ,

$$\Phi = -\frac{2k}{\sqrt{\rho\rho'}}\,\mathrm{K} + \frac{4}{k\sqrt{\rho\rho'}}\mathrm{K} - \frac{4}{k\sqrt{\rho\rho'}}\,\mathrm{E}$$

or

$$\Phi = \frac{2}{\sqrt{\rho\rho'}}\left\{\left(\frac{2}{k}-k\right)\mathrm{K} - \frac{2}{k}\,\mathrm{E}\right\}$$

and consequently

$$\mathrm{P} = \frac{1}{\pi}\int\int\lambda'\sqrt{\frac{\rho'}{\rho}}\left\{\left(\frac{2}{k}-k\right)\mathrm{K} - \frac{2}{k}\,\mathrm{E}\right\} d\rho'dz',$$

$$\mathrm{U} = -\frac{1}{\pi}\int\int\lambda'\sqrt{\frac{\rho'}{\rho}}\left\{\left(\frac{2}{k}-k\right)\mathrm{K} - \frac{2}{k}\,\mathrm{E}\right\} \sin\vartheta\, d\rho'dz',$$

$$\mathrm{V} = \frac{1}{\pi}\int\int\lambda'\sqrt{\frac{\rho'}{\rho}}\left\{\left(\frac{2}{k}-k\right)\mathrm{K} - \frac{2}{k}\,\mathrm{E}\right\} \cos\vartheta\, d\rho'dz'.$$

In the function that we have denoted by Φ we see that the derivatives taken for z and z' have the same absolute values but opposite signs; consequently

$$\int\int\lambda\lambda'\frac{d\Phi}{dz}\,\rho\rho'\,d\sigma\,d\sigma' = o$$

when $d\sigma = d\rho dz$. But we have also

$$\frac{d\mathrm{P}}{dz} = \frac{1}{2\pi}\int\lambda'\rho'\frac{d\Phi}{dz}d\sigma'$$

therefore

$$\int\rho\frac{d\mathrm{P}}{dz}\lambda d\sigma = o.$$

But we have for the energy T the equation

$$\mathrm{T} = \int d\tau(\mathrm{U}\xi + \mathrm{V}\eta)$$

Substituting for U, V, ξ and η their values this becomes,

$$\mathrm{T} = \int\mathrm{P}\lambda\,d\tau = \iiint\mathrm{P}\lambda\,\rho d\rho dz d\vartheta$$

integrating with respect to ϑ from o to 2π, and writing again $d\rho dz = d\sigma$,

$$\mathrm{T} = 2\pi\int\mathrm{P}\rho\lambda d\sigma.$$

Substituting for P its value we have

$$\mathrm{T} = \int\Phi\rho\rho'\lambda\lambda'\,d\sigma\,d\sigma'$$

when of course $d\sigma$ and $d\sigma'$ denote the cross sections of the vortex filaments under consideration. Let S denote the component of velocity in which ρ increases:

$$\mathrm{S} = \sqrt{u^2+v^2}$$

which is the same as

$$u = \mathrm{S}\cos\vartheta \qquad v = \mathrm{S}\sin\vartheta.$$

But

$$u = -\frac{d\mathrm{V}}{dz} = -\frac{d\mathrm{P}}{dz}\cos\vartheta$$

$$v = \frac{d\mathrm{U}}{dz} = -\frac{d\mathrm{P}}{dz}\sin\vartheta$$

therefore

$$\mathrm{S} = -\frac{d\mathrm{P}}{dz}.$$

Now for w we have

$$w = \frac{d\mathrm{V}}{dx} - \frac{d\mathrm{U}}{dy}$$

and substituting for U and V their values.

This gives

$$w = \frac{d\mathrm{P}}{d\rho} + \frac{\mathrm{P}}{\rho}$$

or

$$w\rho = \frac{d(\mathrm{P}\rho)}{d\rho}$$

and we may also write

$$s\rho = -\frac{d(\mathrm{P}\rho)}{dz} \text{ since } \frac{d\rho}{dz} = o.$$

From these we have the equation $\int\rho\frac{d\mathrm{P}}{dz}\lambda\,d\sigma = o$ in the form

$$\int\rho s\lambda d\sigma = o$$

and also

$$s=\frac{d\rho}{dt} \text{ and } w=\frac{dz}{dt}$$

therefore

$$\int \rho \frac{d\rho}{dt} \lambda\, d\sigma = o$$

and since for each vortex filament $\lambda d\sigma$ is constant this gives

$$\int \rho^2 \lambda d\sigma = \text{const.}$$

Some other interesting forms may be given before we proceed to the examination of a special case. We had

$$k^2=\frac{4\rho'\rho}{(z'-z)^2+(\rho'+\rho)^2}$$

Taking logarithms this becomes

$$2 \log k = \log \rho\rho' - \log [(z'-z)^2+(\rho'+\rho)^2]$$

Differentiating with respect to ρ and z we obtain

$$\frac{2}{k}\rho \frac{dk}{d\rho}=\frac{(z'-z)^2+\rho'^2-\rho^2}{(z'-z)^2+(\rho'+\rho)^2}$$

$$\frac{2}{k} z \frac{dk}{dz} = \frac{zz(z'-z)}{(z'-z)^2+(\rho'+\rho)^2}$$

consequently,

$$\frac{2}{k}\left\{\rho \frac{dk}{d\rho}+z\frac{dk}{dz}\right\}=\frac{z'^2-z^2+\rho'^2-\rho^2}{(z'-z)^2+(\rho'+\rho)^2}$$

The denominator of this second member does not change by the interchange of accented and unaccented letters, but the numerator does change its sign, also k does not change by making the same transfer, therefore

$$\rho\frac{dk}{d\rho}+z\frac{dk}{dz}$$

assumes the opposite value by writing the accented letters in the place of the unaccented and vice versa. If from the value of Φ in terms of K and E we obtain $\frac{d\Phi}{dk}$ we will see that this quantity does not alter by the interchange of accented and unaccented letters, consequently the quantity

$$\frac{d\Phi}{dk}\left(\rho \frac{dk}{d\rho}+z\frac{dk}{dz}\right)$$

assumes the opposite value after the interchange. We have by partial differentiation of Φ

$$\frac{d\Phi}{d\rho}=\frac{d\Phi}{d\rho}+\frac{d\Phi}{dk}\frac{dk}{d\rho}$$

$$=\frac{d\Phi}{dk}\frac{dk}{d\rho}-\frac{1}{z}\frac{\Phi}{\rho}$$

$$\frac{d\Phi}{dz}=\frac{d\Phi}{dk}\frac{dk}{dz}$$

Consequently the above quantity assumes the form

$$\rho\frac{d\Phi}{d\rho}+z\frac{d\Phi}{dz}+\frac{1}{z}\Phi$$

And by virtue of the property proved for this quantity we know that

$$o=\int\int\left(\rho\frac{d\Phi}{d\rho}+z\frac{d\Phi}{dz}+\frac{1}{z}\Phi\right)\rho\rho'\lambda\lambda'\, d\sigma d\sigma'$$

But we have

$$\mathrm{P}=\frac{1}{2\pi}\int\int \lambda'\rho' d\rho' dz' \Phi$$

therefore

$$\int\left(\rho\frac{d\mathrm{P}}{d\rho}+z\frac{d\mathrm{P}}{dz}+\frac{1}{z}\mathrm{P}\right)\rho\lambda d\sigma = o$$

Since now

$$\rho\frac{d\mathrm{P}}{d\rho}=\frac{d(\mathrm{P}\rho)}{d\rho}-\mathrm{P}$$

and

$$w\rho=\frac{d(\mathrm{P}\rho)}{d\rho}, \quad s=-\frac{d\mathrm{P}}{dz}$$

this equation becomes

$$\int(w\rho-zs)\rho\lambda d\sigma-\frac{1}{z}\int \mathrm{P}\rho\lambda d\sigma = o$$

or

$$\int(w\rho-zs)\rho\lambda d\sigma=\frac{\mathrm{T}}{4\pi}.$$

We will now introduce the comple mentary modulus k' defined by

$$k'^2+k^2=1$$

we have for this modulus

$$k'^2=1-k^2=\frac{(z'-z)^2+(\rho'-\rho)^2}{(z'-z)^2+(\rho'+\rho)^2}$$

If k is very nearly equal to unity, that is, if k' is indefinitely small we see that Φ will be of the same order of magnitude as K, and again, that P will be of the same order as Φ. We will examine the order of K on the supposition that k' is infinitely small. Since k' is indefinitely small, we have, neglecting higher powers than the second,

$$k=\sqrt{1-k'^2}=1-\frac{1}{2}k'^2$$

Now

$$\mathrm{K}=\int_0^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{1-k^2\sin^2\psi}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{1-\sin^2\psi+k'^2\sin^2\psi}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{\cos^2\psi+k'^2\sin^2\psi}}$$

or, by introducing an indefinitely small quantity ε which is, nevertheless, indefinitely large as regards k',

$$K = \int_0^{\frac{\pi}{2}-\varepsilon} \frac{d\psi}{\sqrt{\cos^2\psi+k'^2\sin^2\psi}} + \int_0^{\frac{\pi}{2}-\varepsilon} \frac{d\psi}{\sqrt{\cos^2\psi+k^2\sin^2\psi}}$$

Write now in the first integral $\frac{\pi}{2}-\theta$ in in place of Ψ: since throughout the integral θ is small the integral becomes

$$\int_0^{\varepsilon} \frac{d\theta}{\sqrt{k'^2+k^2\theta}} = \frac{1}{k}\log\frac{k\varepsilon+\sqrt{k'^2+k^2\varepsilon^2}}{k'}$$

or since k' is infinitely small with regard to $k\varepsilon$ this is $=\frac{1}{k}\log\frac{2k\varepsilon}{k'}$ or $=\log\frac{2\varepsilon}{k'}$.

In the second integral $k'\sin\psi$ is throughout small as regards $\cos\psi$ and this integral is

$$\int_0^{\frac{\pi}{2}-\varepsilon} \frac{d\psi}{\cos\psi} = \log\tan\left\{\frac{1}{2}\pi-\frac{1}{2}\varepsilon\right\}$$

or what is the same thing $=\log\frac{2}{\varepsilon}$. Hence we have

$$K = \log\frac{2\varepsilon}{k'} + \log\frac{2}{\varepsilon} = \log\frac{4}{k'}$$

Consequently for k' indefinitely small we see that K is indefinitely large, and Φ and therefore P are indefinitely large of the order $\log k'$. Of course Φ does not depend on E as for k nearly equal to unity we have,

$$E = \int_0^{2\pi} \cos\psi\, d\psi = 1.$$

Representing the elliptic integral of the first kind as is usual by F, we have

$$F = \int_0^{\psi} \frac{d\psi}{\sqrt{1-k^2\sin^2\psi}}$$

and also

$$E = \int_0^{\psi} (1-k^2\sin^2\psi)^{\frac{1}{2}}\, d\psi.$$

Differentiating for k, and writing $\sqrt{1-k^2\sin^2\psi}=\Delta$

$$\frac{dF}{dk} = \int \frac{k\sin^2\psi d\psi}{\Delta^3}$$

$$\frac{dE}{dk} = -\int \frac{k\sin^2\psi d\psi}{\Delta}$$

Now writing $\sin^2\psi = \frac{1}{k^2}(1-\Delta^2)$ we see that these two integrals depend on

$$\int\frac{d\psi}{\Delta},\quad \int\Delta d\psi,\quad \int\frac{d\psi}{\Delta^3}:$$

the two first of these are F and E respectively; as regards the third, we have

$$\frac{d}{d\psi}\cdot\frac{\sin\psi\cos\psi}{\Delta} = \frac{1-2\sin^2\psi+k^2\sin^4\psi}{\Delta^3}$$

or

$$k^2\frac{d}{d\psi}\cdot\frac{\sin\psi\cos\psi}{\Delta} = \frac{\Delta^4-k'^2}{\Delta^3} = \Delta-\frac{k'^2}{\Delta^3}$$

and thence by integration,

$$\int\frac{d\psi}{\Delta^3} = \frac{1}{k'^2}E - \frac{K^2\sin\psi\cos\psi}{k'^2\Delta}$$

The expressions for $\frac{dE}{dk}$, and $\frac{dF}{dk}$, thus become,

$$\frac{dF}{dk} = \frac{1}{kk'^2}\left\{E-k'^2F\right\} - \frac{k\sin\psi\cos\psi}{dk'^2}$$

$$\frac{dE}{dk} = \frac{1}{k}\left\{E-F\right\}$$

and for the complete functions when $\psi=\frac{\pi}{2}$,

$$\frac{dE}{dk} = \frac{1}{kk'^2}[E-k'2F]$$

$$\frac{dE}{dk}=\frac{1}{k}[E-F]$$

When k' is indefinitely small the first of these is of the order $\frac{1}{k'^2}$ and the second by k'. Now we have obviously if k' be indefinitely small that $\frac{d\Phi}{dk}$ is of the order $\frac{1}{k'^2}$. We had also

$$\frac{dk}{dz}=\frac{k(z'-z)}{(z'-z)^2+(\rho'+\rho)^2}$$

$$\frac{dk}{d\rho}=\frac{k}{2\rho}\frac{(z'-z)^2+\rho'^2-\rho^2}{(z'-z)^2+(\rho'+\rho)^2}$$

These quantities are of the same order as k'. Therefore

$$\frac{d\Phi}{d\rho}=\frac{d\Phi}{dk}\frac{dk}{d\rho}\frac{1}{z}\frac{\Phi}{\rho}$$

and

$$\frac{d\Phi}{dz}=\frac{d\Phi}{dk}\frac{dk}{dz}$$

are of the same order as $\frac{1}{k'}$. By the aid of these preliminary investigations we will now proceed to the examination of the case when only one vortex ring exists in the fluid, and will further more suppose this ring to be of indefinitely small cross section and of the same order of magnitude as the indefinitely small quantity ε. We may again write as before

$$m=\int\lambda d\sigma$$

as m will be finite and as $d\sigma$ is of the order ε, λ must be of the order $\frac{1}{\varepsilon^2}$. Assume the equation of a circle such that the fluid elements of which it is composed shall lie indefinitely near the vortex filament. Let its equations be

$$\rho=\rho_0 \qquad z=z_0.$$

We had

$$P=\frac{1}{2\pi}\int\int\lambda'\rho'd'\rho dz'\Phi=\frac{m}{2\pi}\rho\rho_0\Phi$$

$$(z-z_0)\rho\rho_0)$$

for all points lying at a finite distance from the circle. For these points by aid of the equations

$$\Phi=\frac{z}{\sqrt{\rho\rho_0}}\left\{\left(\frac{2}{k}-k\right)K-\frac{z}{k}E\right\}$$

$$\rho s=-\frac{d(P\rho)}{dz},\ w\rho=\frac{d(P\rho)}{d\rho}$$

we can find the values of s and w. But a difficulty exists inasmuch as ρ_0 and z_0 are functions of the time and as such will have to be determined, and to that end, it is necessary to consider the points that lie indefinitely near the circle, or points in the vortex filament.

Suppose that the two circles $(\rho, z), (\rho_0, z_0)$ are at a distance apart, that is of the same order as ε. If we call r the distance between them, we have

$$r^2=\rho^2+\rho_0^2-2\rho\rho_0\cos\vartheta+(z_0-z)^2$$

or since ϑ is indefinitely small

$$r^2=(\rho-\rho_0)^2+(z-z_0)^2$$

this is of the same order as k'^2 therefore k' is of the same order as ε—therefore by our preceding investigations we see that P is of the same order as Φ when k' is of the same order as ε. And also that P is of the order log. ε. We had for the energy

$$T=2\pi f P\rho\lambda d\sigma$$

therefore T is of the same order as log. ε. The preceding investigations taken in connection with the equations

$$s\rho=-\frac{d(P\rho)}{dz},\quad w\rho=\frac{d(P\rho)}{d\rho}$$

show that inside the vortex ring s and w are of the same order as $\frac{1}{\varepsilon}$. We will now examine more closely ρ_0 and z_0. We have

$$\rho_0^2\int\lambda\sigma=\int\rho^2\lambda d\sigma$$

$$z_0\int\rho^2\lambda d\sigma=\int z\rho^2\lambda d\sigma$$

We assume that λ is constantly of the same sign, consequently the circle (ρ_0, z_0) lies either in or indefinitely near the vortex filament.

Now we found that, $\lambda d\sigma$ not varying with the time,

$$\int\rho^2\lambda d\sigma=\text{const.}$$

consequently ρ_0 does not vary with the time. That is, if only one vortex filament exists in the fluid its radius will remain unaltered, during the motion. Now to examine z_0. We have the equation

$$z_0\int\rho^2\lambda d\sigma=\int z\rho^2\lambda d\sigma$$

$$\int\lambda d\sigma=m$$

from these

$$m_o\rho^2 z_o = \int z\rho^2 \lambda d\sigma$$

Differentiating with respect to t

$$m\rho_o^2 \frac{dz_o}{dt} = \int \rho^2 \frac{dz}{dt} \lambda d\sigma + 2\int z\rho \frac{d\rho}{dt} \lambda d\sigma$$

But we had also

$$\frac{1}{4\pi} T = \int (w\rho - sz)\rho\lambda d\sigma$$

$$= \int \rho^2 \frac{dz}{dt} \lambda d\sigma - \int \rho^2 \frac{d\rho}{dt} \lambda d\sigma$$

$$= m\rho_o^2 \frac{dz_o}{dt} - 3\int z\rho \frac{d\rho}{dt} \lambda d\sigma$$

consequently

$$m\rho_o^2 \frac{dz_o}{dt} = \frac{1}{4\pi} T + 3\int z\rho \frac{d\rho}{dx} \lambda d\sigma$$

T, we have seen, is constant and infinitely great, of the same order as $\log \varepsilon$; the difference between the values of z, in the second member, we know to be of the same order as ε, and we have seen that $\frac{d\rho}{d\tau}$ is of the order of $\frac{1}{\varepsilon}$; consequently the second member of the right hand side of the equation is finite. Therefore, it follows that $\frac{dz_o}{dt}$ is infinitely great of the order of $\log \varepsilon$, and, neglecting the finite term — is constant. Since T is positive, $\frac{dz_o}{dt}$ must have the same sign as m, *i. e.*, as λ. Thus, if only one vortex ring exist in the fluid, it will retain its radius unaltered during the motion, and will advance in the direction of the axis of z with the velocity $\frac{dz_o}{dt}$. Now, as this vortex motion implies motion in all the particles of the fluid we have, that all the fluid particles at a finite distance from the filament flow through the ring in the direction of z, or the reverse, according to the sign λ. If $\Theta = o$, we have $\lambda = \eta$, and, according to the convention for positive rotation we have, that the motion in the direction of z will be positive, if, in the case $\theta = o$, η is positive. Therefore, it follows that the ring is moving in the same direction as the fluid particles are flowing. I will now give the concluding remarks of Helmhotz's great memoir as nearly as may be. We can now readily see in general how two vortex rings having the same axis will move with reference to each other, by observing that each will have its motion modified, due by the motion of the particles of fluid, due to the rotation of the other. Suppose that both rings have the same direction of rotation, then they will both move forward in the same direction, and the former will widen and move more slowly, while the latter will contract and move forward more rapidly, finally overtaking and passing through the former, when the same operation will be repeated, the rings continually changing position throughout the motion.

Suppose that the vortex rings have equal radii, the result is not changed in the case of the same direction of rotation existing for both. But now let them equal radii, and equal but opposite angular velocities; they will approach each other, and both will expand—approaching very near the effect of one upon the other is greatly increased—and they expand with constantly increasing velocity.

Now suppose that the rings having equal and opposite angular velocities, are symmetrical to each other. Then the motion in the direction of the axis of those particles that lie midway between the rings is o. We can conceive this surface in which these particles lie as a fixed boundary, and we have the case of vortex rings moving in contact with a fixed surface. These rings can be readily formed in water; or, rather, half rings can be formed, if we draw through the water rapidly, and for a short distance, a half immersed hemispherical vessel. Half rings will be formed in the water, having their axes in the fine surface of the fluid, which will move exactly as described in the theory. The free surface of the water will form a limiting plane, passing through the axes of the rings, and will not affect the motion. Rings of tobacco smoke have a rapid motion forwards in the direction of and due to the impulsive force which produced them; at the same time the ring flows through itself in the direction of the motion of translation.

It is very interesting to observe the motions of smoke rings, and for this purpose the following simple apparatus, which has been described in a great many places, will be found useful: A rough box, about ten inches long, and the same

height and width, is large enough; one end of the box to be open, and over this stitch a piece of cloth or rubber; make a hole, about three inches in diameter, in the opposite end of the box, and a number of slides having smaller holes in them, to be placed over the larger opening and concentric with it. Now place inside of the box a vessel containing salt, on which pour strong sulphuric acid; and also place in the box a piece of cotton saturated with ammonia; fumes of ammonium chloride will immediately fill the box. Now tap on the stretched membrane; rings will issue from the hole in the slide at the opposite end, and will move forward with velocities proportional to the force of the blow struck. A very light tap is all that is necessary, and, indeed, is all that can be given, if it is desired to investigate the motion, as, otherwise, the rings move forward with such velocity that they can scarcely be followed with the eye. If the rings are allowed to impinge upon a surface, the rotational velocity is suddenly increased very much, and the rings thus spread out over the surface.

The same effects will be noticed if two rings be allowed to meet each other in their motion through the air. If the orifice be elliptic, the rings will be seen to interchange rapidly their axes, vibrating about a mean circular position.

If bubbles of phosphuretted hydrogen be allowed to escape into the air, each bubble, as it breaks, forms a vortex ring of phosphoric anhydride, which is composed of a number of small rings.

The reader is advised to read, on the subject of vortex motion, Sir William Thomson's paper in the Edinburgh Transactions for 1869; also an article by D. Bobylew, in the *Mathematische Annalen*, Vol. VI., in which he shows that the equation $\omega k =$ const. is true not only for frictionless fluids, but also for those in which the friction has to be taken into account.

The following articles will also be found to contain much of interest: On the Motion of Water in a Rotating Rectangular Prism, A. G. Greenhill, *Quarterly Journal of Mathematics* for Nov., 1877; on Plane Vortex Motion, by the same author, and in the June number of the same journal for 1877. There are also several interesting articles in the *Messenger of Mathematics* for the year 1878, notably one of vortex motion in elliptic cylinders; and on the motion of a liquid in a rotating quadrantal cylinder.

SANITARY WARNINGS FROM HIGH PLACES.

From "The Builder."

In the destructive course of the cholera through India, it was remarked by some observers that the pestilence seemed to come from below. Not only were the inmates of houses at the foot of a hill attacked sooner than those towards the summit, but the fowls and other small animals, naturally abiding on the surface of the ground, sickened and died some days before dangerous symptoms broke out in the human inhabitants. If this statement be thoroughly reliable, it is of no little value, both as affording a warning, and as indicating in what direction the sanitary engineer should look in his precautions against the danger intimated. Exhalation, in one word, is, at all events, one sign or cause of danger, in local and epidemic diseases, and the remedy against mischievous exhalation is, not only drainage, but proper regulation of the entire water *régime*. Baking ground, in some cases, has proved even more sickly than swamps.

It is not, however, from the lower levels, either physically or socially regarded, that the most serious warnings to provide against future mischief now come. The remark which has more than once been made in our columns of the mode in which that subtle and dangerous malady which now takes the form of diphtheria, and now that of enteric or typhoid fever, chooses its victims among the very flower of society, has received fresh illustration. In the past few weeks fatal blows have been struck in two royal houses, in that of the Prince Royal of

Prussia and in that of the Duc de Montpensier. And the alarm thus caused is intensified by the recollection how many of those who could so ill be spared have been carried off within the last few years, from the palace and from the castle, by a disease which we profess to believe is preventible by the engineer.

It is to be hoped that the true lesson will be drawn from these repeated appearances of what we must regard as malarious diseases in the homes of luxury. It would not be unworthy of the Society of Arts to direct a special investigation to the subject. If we venture on a few hints, it is rather as indicating the line of inquiry that the public welfare seems to demand, than as at all anticipating the result of an inquiry of which the need has become very urgent.

Predisposition to attacks of this nature may prove to be either personal or local. It is not clear, but it may be suspected, that there is some element at work, in the present state of civilization, which renders the more gently nurtured, or more highly cultured, members of society specially unfitted to resist malarious influences. Connected with this must be borne in mind the manner in which the external atmosphere is more and more kept out from our houses. Doors and windows close better, draughts are more carefully excluded, than of old. Appliances are introduced for artificially warming the passages and vestibules, the natural function of which places is to afford a graduated transition from the warm atmosphere of a chamber to the external temperature. Clothing is much more complex than was formerly the case. In the time of our grandfathers a man was called a puppy if he wore an overcoat. What would those hardy gentlemen have said to the "Ulsters" of the present day? or the sealskin jackets and coats? Human habit is so much modified by circumstances, that the adoption of all these safegards against an occasional chill may have a direct tendency to lower the resisting power of the constitution. And there are well known facts that square with this view. Such is the influence on the constitution of the prolonged heat of tropical or sub-tropical countries. Even in Italy, where the thermometer rarely rises above 96° Fahr. in the middle of the summer, this effect is more perceptible. Those who can fly to mountain heights or cool spots invariably do so for about six weeks in the summer; those who cannot do so (we now speak of foreigners resident in Italy) disregard the first hot season, and laugh to see the natives slowly slinking from shade to shade, and never moving without an umbrella,—truly so called, when used not against rain, but against sun. The second season, however, becomes more serious. The third still more so; and a succession of summers, even as far north as Naples, is a very severe trial to an Englishman or a German. The body seems to dry up, the hair to become like hay, and old age to advance with untimely speed. No one who has had experience of the matter will deny that this is the general rule. The inference is not unnatural that the greater comfort, as we regard it, at all events the more sustained heat, which we are steadily giving to our abodes, is really tending to lower our constitutional power of resistance, not only to the great tonic, cold, but to those influences against which that tonic has the prime function of strengthening the frame.

When we note, further, that it is so often in the palace and the noble mansion that the outburst of the fatal malady occurs, this view of the case is yet further supported. The first point, therefore, for the inquiry of the scientific physician is, how far do greater habits of luxury, avoidance of cold, late hours, gas, carriage exercise in lieu of walking or of riding, and luxury of food, sap and undermine the vigor of the individual, and thus, in an increasing ratio, the vigor of the race? And if this prove to be the case, is there no Lycurgus to be found to raise the cry of "Health before ease and luxury?"

The second point regards the sanitary condition of our houses themselves. This is a different thing from the effect of luxury on the frame. It may be possible, or at all events conceivable, for a great house to be so built, and so tended, that a given temperature, let us say 66° Fahrenheit, may be constantly maintained through vestibules, corridors, staircases, and rooms; and at the same time that the ventilation shall be such as to change the air with sufficient rapidity, and to prevent the entrance of any poisonous gas. But in our variable

climate the cost of the provisions for this purpose would be great, and the necessity for constant supervision would demand attendants of a high degree of education to be detailed for this special service. As it is, warmth is too frequently secured at the cost of pure air. The products of the combustion of gas, and the minute quantities of gas (where this mode of lighting is allowed in sitting or sleeping room) that escape unburnt, either from the pipes, the fittings, or the burners, are constantly vitiating the atmosphere of the house. Our own experience is that plants will never thrive in a room in which gas is burnt. If such be the effect on the vegetable tissue, what must be the case with the more delicate tissue of the lungs? We know houses where sore throat is frequent, almost constant, during the winter, and from which we believe that it might be banished by cutting off the gas, and having the windows regularly opened in all weathers.

Then there is the great danger attendant on the water-closet system. We are not about to propose any Utopian remedy. We do not deny that in this system we have, all things considered, the best yet discovered for the removal of the *debris* of organic change from large cities. But it will not do to shut our eyes to the fact that the system is attended with great danger. The slightest irregularity—stoppage, leakage, frost, want of water, too much water, incurs the risk of turning a fatal exhalation of sewer gas into the house. Sometimes this arises from external causes alone, quite beyond the control of the residents. An irruption of gas may occur in offensive quantities. This, horrid as it is, is far less dangerous than a more subtle and gradual escape. Sometimes it is the very effort to ventilate the sewer that provides a stream of fatal gas to dribble down an unused chimney, or to lurk under the eaves of the attic stories. Sometimes, and that more especially in large hotels, it is sheer neglect. We know one instance in one of these buildings, which it would not be proper to particularize, of this nature. A child, heir and hope of his family, was staying with his parents at this place for a day or two. The mother discovered the neglected state of one of the closets, to which the child had been taken, and the family left the house forthwith. It was too late; the seeds of the malady had been sown, and diphtheria very soon carried off the only hope of his parents. We speak of one instance within our immediate circle of acquaintances, and within a very few months. How many deaths are due to the like cause? How many, for that matter, to this identical source of mischief?

In referring to this we may, it is true, be reverting to advice formerly given in our own pages. The story of sewer-gas poisoning is not new. But it is none the less important to bring the subject before our readers in its proper connection. We are attempting to account for a very threatening phenomenon. It is one that has for some time been denounced, but that seems to be increasing in virulence, from year to year. The evil must be either essential, or accidental. It must spring either from the increased power of mischief, or from the decreased power of resistance. In our opinion it is due to the combined influences of both causes. Habits of luxury, or, if we may even not go so far as that, habits of delicacy, are, we fear, impairing the constitutions, especially of the richer classes. The provision for those habits is affecting the salubrity of our dwellings. That both these influences are to some extent, at work, we think, is undeniable. Is it not high time that the matter should be looked in the face?

With regard to the question of house ventilation, the scheme has recently been put forward of a large and lofty chimney, connected with a block of houses, which by its draught should sweep and purify the entire block. There is something very fascinating in this idea, especially to those who are practically aware of how well chemical works can be kept pure by a lofty chimney. It may be well to look, for a few minutes, at the leading facts that would determine the size and number of such chimneys. We lately gave a brief account of the highest chimney in the world, one of 468 feet high, at the Port Dundas works, Glasgow. The circular area of the tube of this chimney, at the top, is under 88 square feet. This chimney was built with a taper, the propriety of which is questionable, as, in order to utilize the larger part of the diameter a higher velocity is requisite

through the narrower part of the chimney, while the disposition of the air will be to lose velocity as it cools. But, taking the 88 feet of area, and allowing a velocity of 33 feet per second (which is equal to 22½ miles per hour, or that of a stiff gale) to be attained, 118,800 cubic feet of air would pass through the aperture per minute. As a check on that calculation we take from Mr. D. K. Clark's valuable "Manual of Rules, Tables, and Data for Mechanical Engineers" (p. 927), the maximum quantity of air discharged per minute by a Guibal fan, working at Staveley Colliery, of 30 feet diameter and 10 feet wide, which is 110,005 cubic feet per minute. To attain the speed of 33 feet per second, a pressure of 15.5 lb. per square foot must be maintained. The quantity of atmospheric air required for the consumption of one ton of coal, of average quality, is 313,600 cubic feet; so that a chimney of the area indicated, provided such a draught could be maintained, would effect the consumption of a ton of coal in a little under three minutes, or 22 tons of coal per hour. Taking eighteen hours out of the twenty-four for the combustion, we obtain close upon 400 tons per day, or 146,000 tons of coal per annum. If we allow a ton of coal per unit of the population per annum, this would require from twenty-four to thirty such chimneys for the whole of London.

But when we come to look at the question of ventilation, for the purpose of respiration, it will at once be seen how inefficient this powerful draught would be to discharge the functions of our present wasteful and inconvenient chimneys. According to Mr. Bailey Denton, in his first letter on sanitary engineering (p. 2), 23,000 cubic feet of ordinary air are required per individual per twenty-four hours in order to keep the vitiation of the air below 1 per 1,000, at which point impurity becomes perceptible to the nose. Allowing only eighteen hours to be spent within doors, either in bedrooms, houses, or offices, we shall find that the 824 million cubic feet passing through the hypothetical chimney in that time would only provide for the ventilation requisite for a little over 47,000 human beings. Three times as much ventilating power, therefore, is required for keeping the chambers of a house pure, as is requisite for maintaining combustion of coal at the allowance of one ton per inhabitant per annum. And no provision is made for the products of combustion of gas or other artificial lights.

We need not add together the two quantities; as the partially vitiated air would to a great extent serve for combustion. But, all things considered, it would seem that a chimney possessing as high a power of draught as we have named would be requisite for, at least, from every 33,000 to 40,000 of population, supposing the whole of the ventilating arrangements to be brought under proper control by its means. As to the details of these arrangements, we cannot at present find space for discussion. But it may be mentioned that from 400 feet to 740 feet per minute is the rate of ventilation which is effected in mines; the upcast pit having the former rate of draught, and the velocity in the several channels varying inversely as their areas. The point where the above considerations practically bear on our present subject is this. The removal of air from a house that is requisite to ensure a due ventilation for sanitary purposes is so much more considerable than the removal which is due to, and effected by, the combustion of the amount of fuel proportioned to the number of inhabitants in the dwelling, that special attention should be directed to the subject in all large houses. Chiefly is this the case in those houses, whether large or small, in which it is the object of the housekeeper to maintain a high, or a moderate but invariable, temperature, through all parts of the house, in the winter season. This can only be done, with safety, by warming the air as it enters; and the question is, whether either this or any other mode of keeping up the temperature of the vestibules and corridors is not a direct invitation to disease. We suggest that it is so, in two modes. First, that the resisting power of the frame is lowered by the maintenance of an invariable temperature. While persons remain within the house, the aëration of the blood and the animal vigor will be abated; and on going out the cold will be more keenly felt, and very possibly more injuriously felt, than if the passages had been cool. The other,

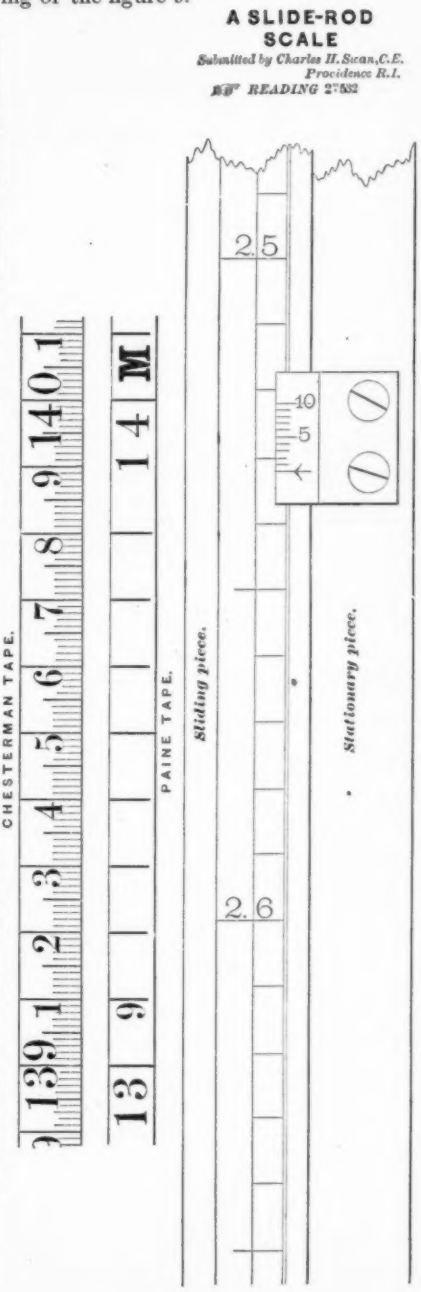
and entirely independent question, is that of the risk that there is in all highly warmed houses of slighting the proper ventilation. The carbonic acid exhaled from the lungs is, of course, so much direct poison, unless removed. To this have to be added all other sources of impurity, products of combustion, gas resulting from the course of the organic functions, emanations from the pet birds or other animals, and, most dangerous of all, infiltration of sewer gas. A wholesome thorough draught is the only safeguard against this domestic malaria; and very few are the instances in which this is to be ensured without the utmost regularity and care as to freely-opened windows. Those who know what Belgian, German, and even French hotels are in this respect, will be likely to form the opinion that palaces and noble mansions may be made, and very frequently are made, far less safe and salubrious, especially for the young, than were the rude and draughty nooks in the halls and castles of our ancestors, or than our very modest cottages in the present day.

THE GERMAN NAVY.—The *personnel* of the German Navy is to be increased this year by 29 officers and 343 warrant officers, petty officers and men, although, according to the original plan for the establishment of the fleet, the increase should comprise only 168 officers and men of all ranks. The engineer department is also to be augmented by 95 petty officers and men, instead of but 73, the number prescribed in the plan. Altogether, since 1875, more than 1000 men have been added to the strength of the divisions of seamen, so that the German Navy now comprises 419 sea officers, 24 engineers, 5459 warrant officers, petty officers and men in the two divisions of seamen, besides 1001 petty officers and men in the engineer department. Of the two divisions of seamen, one belongs to each of the two naval stations, the Baltic and North Sea, the headquarters of the one being at Kiel, and of the other Wilhelmshafen. Each division is commanded by a post captain, and is divided into five detachments, one of which is composed solely of seamen gunners. Besides these divisions of seamen, there is also a "dockyard division" at the headquarters of each station, Kiel and Wilhelmshafen.

REPORT ON THE METRIC SYSTEM (ERRATA).—A couple of errors in the engraving on page 382 of our last issue are corrected in this cut.

The reading of the slide rod should have been 2.532, as is here given.

The numbering on the Paine tape was incorrectly represented by the misplacing of the figure 9.



REPORTS OF ENGINEERING SOCIETIES.

AMERICAN SOCIETY OF CIVIL ENGINEERS.—The Eleventh Annual Convention of the Society will be held at Cleveland, beginning Tuesday, June 17th, 1879.

Sessions for the consideration of professional subjects, and one for the transaction of business will be held.

The details of the programme will be announced as soon as determined by the local committee.

The following is a list of topics to be considered with reference to papers published in Transactions during the preceding year:

American Engineering at International Exhibitions.—CLXXIV. American Engineering as illustrated at the Paris Exposition of 1878. George S. Morison, Edward P. North and John Bogart.

Bridges.—Discussion of Paper CXL. The Determination of Stresses in the Eye-Bar Head. De Volson Wood. Vol. VII, page 189. Discussion of Papers CXLIV and CXLIX. Relative Quantities of Material in Bridges of different kinds, of various heights. William H. Searles. Vol. VII., page 192.

Cements.—Discussion on Cements. Don J. Whittemore. Vol. VII, page 274. Discussion on Cements and Strength of Bricks. F. Collingwood. Vol. VII., page 280.

Foundations.—CLXXII. The use of Compressed Air in Tubular Foundations, and its application at South Street Bridge, Philadelphia, Pa. D. McN. Stauffer.

Hydraulics.—CLX. On the cause of the Maximum Velocity of Water flowing in Open Channels being below the Surface. James B. Francis. CLXI. The Flow of Water in Pipes under Pressure. Charles G. Darrach. Discussion on the Cause of the Maximum Velocity of Water flowing in Open Channels being below the Surface, and also on the Flow of Water in Pipes under Pressure. Theo. G. Ellis, C E. Emery, Clemens Herschel, De Volson Wood and John T. Fanning. Vol. VII., page 122. CLXVII. Distribution of Rain-fall during the great storm of October 3d and 4th, 1869. James B. Francis. CLXVIII. The Gauging of Streams. Clemens Herschel. CLXXV. The Flow of Water in Small Channels, after Ganguillet and Kutter, with Kutter's Diagram modified, and Graphical Tables with special reference to Sewer Calculations. R. Hering.

Masonry.—CLXX. Brick Arches for Large Sewers. R. Hering. Discussion on Brick Arches for Large Sewers. E S. Chesbrough, W. Milnor Roberts, R. Hering and F. Collingwood. Vol. VII., page 258. CLXXI. Fall of Western Arched Approach to South Street Bridge, Philadelphia, Pa. D. McN. Stauffer. Discussion on Nomenclature of Building Stones and Stone Masonry. J. Foster Flagg, J. J. R. Croes, J. P. Davis, F. Collingwood, J. Veazie and E P. North. Vol. VII., page 284.

Metals.—CLXIII. On a newly discovered relation between the Tenacity of Metals and their resistance to Torsion. Robert H. Thurston. CLXIV. Observations on the Stresses developed in Metallic Bars by Applied Forces. Theodore Cooper.

Preservation of Timber.—CLXXVI. The Permanent Way of Railways in Great Britain and Ireland, with special reference to the use of Timber, preserved and unpreserved. Compiled from information received from Engineers in charge of these Railways John Bogart.

Railroads—CLIX. On the Theoretical Resistance of Railroad Curves. S. Whinery. Discussion on the Resistance of Railroad Curves. O Chanute, Chas. E. Emery, E. Yardley, E. P. North, C. L. McAlpine, F. Collingwood and Wm. H. Paine. Vol. VII., page 97. CLXVI. Reminiscences and Experiences of Early Engineering Operations on Railroads, with especial reference to Steep Inclines. W. Milnor Roberts. Discussions on Inclined Planes for Railroads. O. Chanute and William H. Paine. Vol. VII., page 216.

Rivers and Harbors—CLXII. The South Pass Jetties. Descriptive and Incidental Notes and Memoranda E L Corthell. Discussions on the South Pass Jetties. Charles W. Howell, E. L. Corthell, C. Shaler Smith, J. Foster Flagg. Vol. VII., page 159. CLXIX. The Dangers threatening the Navigation of the Mississippi River and the Reclamation of its Alluvial Lands. B. M. Harrod.

Steam Engines.—CLXV. Cushioning the Reciprocating Parts of Steam Engines. John W. Hill. Discussions on Steam Engine Economy. J. Foster Flagg and E D. Leavitt, Jr. Vol. VII., page 194.

Submarine Telephony.—CLXXIII. Submarine Telephoning. Charles Ward Raymond.

In addition to the above papers, it is expected that the following subjects will be presented by papers printed previous to the date of the Convention, or read at its meeting:

Engineering Questions involved in the Development of Electric Lighting. Stephen Chester.

Gelatine Dynamite and High Explosives. Julius H. Striedinger.

Light House Construction. J. G. Barnard

Memoir upon the Construction of the Minot's Ledge Light. B. S. Alexander.

The Construction and Maintenance of Roads. Edward P. North.

The Resistances of Railway Rolling Stock. A. M. Wellington.

The Railroad Crossing of the Allegheny Mountain. Moncure Robinson.

Notes on Early Railroad Engineering. Ashbel Welch.

Remarks on the Causes of Fall of the Western Arched Approach to South Street Bridge, Philadelphia, Pa. J. G. Barnard.

Parabolic Arches in Masonry. W. A. G Emonts.

Notes on the Foundations of Piers of the East River Bridge. F. Collingwood.

Experiments with Cements and appliances for testing. Alfred Noble.

Comparison of Standard Measures, English, French and United States. Arthur S. C. Wurtele.

The Construction of Concrete Blocks at the end of the South Pass Jetties. Max E. Schmidt.

Notes as to construction and operation of the Railroad over the Ratou Mountains, Col., and the construction and performance of the Locomotives thereon. James D. Burr.

Design and Construction Tables for Egg-shaped Sewers. Cyrus G. Force, Jr.

Members of the Society are earnestly requested to furnish information or memoranda upon any of the subjects referred to. They are also invited and expected to take part in the discussions either in person or by sending to the Secretary notes for presentation.

In either case, it will assist the Committee in arranging the details for sessions of the Convention, if members expecting to take part in the discussions will notify the Secretary at once to that effect.

Invitation to visit Pittsburg and the Government Works for the improvement of the river at that place (Davis Island Dam) has been extended by James H. Harlow, member of the Society.

SOCIETY OF ENGINEERS.—At a meeting of the Society of Engineers, held April 21st, Westminster, a paper was read by Mr. J. L. Haddan on "The Essentials which should govern the Construction and Working of Tramways." The author observed that when tramways were first introduced, they were a great advance upon the ordinary roads, but that the modern improvement of roadways had, in the present day, led almost to a reversal of the relative positions of road and tramway. In America, the tramways were superior to the roads, because the latter were sacrified to the former. The tram-rail there, moreover, was available for the moderate ordinary traffic, whilst the speed of the tram service was about twenty per cent. greater than in this country and on the continent. Mr. Haddan alluded to the general tendency to employ wood in roadways, and he described a system of construction by which a road could be made with a perfectly flat surface, and yet be well drained, and which should have the tramway incorporated with it. The tram-rails, he said, would be of wood, and the roadway would be kept surfaced with grit, so that the wood would not form the actual wearing surface. This system of tram and roadway, the author observed, would be homogeneous, and would combine the best possible road for ordinary vehicles, with a perfect tramway for special carriages at less cost than the present method of construction. The author condemned the indiscriminate introduction of railway and omnibus principles into the construction and working of tramways, and described his proposed arrangement for meeting the requirements of a tramway service. This consists of a locomotive engine to be worked by steam and air, the steam being used for compressing, during the journey, its own supply of air as well as that which supplies the continuous motive power for propelling the cars. By reversing, the same driving mechanism acts as a continuous brake, and the same system is so arranged that the driver constantly feels the pull of the train upon a regulator handle. The withdrawal of his hand from this handle is to instantly cause the steam power to block the train. Thus the brake mechanism would always be in action, instead of lying dormant, as in ordinary continuous brakes. The author, in conclusion, stated what, in his opinion, were the technical, physical, and administrative requirements of mechanical traction on tramways generally.

LIVERPOOL ENGINEERING SOCIETY.—At the fortnightly meeting on Wednesday evening, 23rd April, Mr. Arthur J. Maginnis read a paper entitled, "A Few Years' Experience of the Screw Propeller, its Lessons and Results," for which he was awarded the Carlyle Gold Medal at the Institution of Naval Architects, session 1879, as being the best on the subject of screw propulsion. The first part of the paper dealt with the various methods of construction, the materials used for the bosses and blades of large propellers—for which recent experience has proved the old style of solid cast iron to be altogether unsuitable, although still occasionally used owing to their cheapness—and also gave some important particulars of the manner in which failures have occurred from time to time, pointing out how they are to be avoided in the future. The much vexed questions of the relative merits of large and small diameters, and of pitch, uniform, or varying, were next treated in an able manner, and interesting statistics compiled from the logs of some of the most important merchant steamers were given, which showed that moderate reductions of both diameter and pitch had given a reduction of about 8 per cent. in the consumption of fuel on a round voyage, and an increase of about 3 per cent. in speed, the same result being also shown in a marked degree by H. M. S. Iris, the important trials of that ship being also given in a concise table similarly arranged to those of the merchant steamers. Passing over the mode of attachment to the propeller shaft, and the loss occasioned by the corrosion of the blade, etc., the author dealt with the present disputed question of solid cast iron *versus* movable bladed propellers, and pointed out several events, showing the advantages to be greatly in favor of the latter. The various ways by which propellers may become disabled were also noted, some of the most remarkable being illustrated by drawings. After pointing out what experience had taught, as to how a large propeller should be constructed, the important question of the effect of the screw propeller on steering was reviewed at considerable length, and the strange effect produced by the sudden reversing of the propeller, from full speed ahead to full speed astern, was fully treated and explained by explicit diagrams, showing in a lucid manner the directions and flow of the currents of water round about the rudder and screw propeller which produced these effects. Appended to the paper was a neat tabular arrangement of the effect of the propeller on the steering of screw steamers under all circumstances for both right and left-handed propellers, and also a table giving the dimensions and particulars of propellers at present fitted on several of the most important Liverpool steamers.

IRON AND STEEL NOTES.

MANUFACTURE OF STEEL INGOTS.—With a view to economize the labor at present required in producing steel ingots, and also to improve the quality of the ingot by avoiding

the tendency of the metal to boil when run into the moulds, Messrs. Taylor & Wailes, of Panteg, Monmouth, propose to use moulds, which consist of metal troughs placed side by side or parallel to one another, and in rear therof they fit ways on which a traveling trough or gutter runs. This traveling trough is bent at right angles at its forward end to form a spout parallel to the moulds, and it serves to deliver into the moulds the molten metal, which is conducted to it from an inclined stationary gutter or gutters in connection with the furnace. The stationary gutter is so situated with respect to the traveling gutter that the latter will be free to run under the fixed gutter and receive therefrom a continuous supply of the molten metal while it is presenting its discharging mouth to the several moulds in succession. This arrangement not only facilitates the filling of the moulds, but by filling them from the top in a broad stream will remove all tendency which the metal has to boil in the moulds, and will also prevent the cutting of the moulds by the falling metal. The moulds, which will consist of open metallic troughs, may be made of any suitable length, and by an arched block or tile inserted in the moulds the metal poured therein will be divided up to form two ingots.

For facilitating the discharge of these ingots from the moulds they propose to place, transversely of these moulds and near their opposite ends, cross-bars of iron fitted with rough hooks or studs at equal distances apart, and so arranged as to enter the several moulds and bed into the molten metal. These bars will be connected at their opposite ends to a traveling crane, which, when the metal is sufficiently set to secure the connection of the bars with the ingots will be caused to lift the bars, and thereby draw out the ingots from the moulds. The strain thus put on the opposite ends of the castings will cause them to break in the line of the block or tile, and the ingots will then hang perpendicularly in the air, and may be delivered on to any suitable receptacle. In some cases they place a rectangular metal lifting strap with eyes at its extremities inside the moulds, and at such position that it will embrace the ingot to be cast at about the middle of its length. This strap fits close to the inside of the mould, and is covered over with clay, loam, brick, or other substance to prevent the fusion of the metal: this permits of the lifting out of the ingot horizontally by connecting lifting chains with the strap, and when the strap is disengaged from the chain it may readily be separated from the ingot.

In order to prevent any irregularity in the form of the ingot, the ingot mould may be recessed to admit the strap, and the joints may be made good by clay loam. In some cases they propose to make the moulds of the length of the ingot desired, and to set two of such moulds end to end, making good the butt joints by means of clay puddle. The metal cast in such jointed moulds will, when lifted out of the moulds by the means above described, break at the middle of their length or at a point coincident with the line of junction with the moulds, and the ingots will then hang from the lifting bars as explained. This mode of clearing the moulds of the ingots will greatly economize the labor at present required for performing such duty, as the handling of the moulds and the ingots for this purpose is entirely avoided.

RAILWAY NOTES.

RAILWAY TO INDIA.—A paper was read before the Civil and Mechanical Engineers' Society by Mr. Haughton, C.E., on the subject of "A Railway to India."

Mr. Haughton exhibited a map showing the regions to be crossed, taking Constantinople and the town of Shikarpore, on the River Indus, as the terminal points of the railway; he spoke of the suggested Euphrates Valley Railway from Scanderoon or Swadia, on the mainland opposite the horn of the island of Cyprus to the head of the Persian Gulf, with its possible extension along the Mekran coast to Kurrachee, at the mouth of the Indus, as being quite out of the line of the terminal points named, and as being objectionable because of passing through a region destitute of towns after passing Bussorah; such a line, while forming a perhaps useful strategical road to India, entirely failed to tap the larger centers of population and trade of the countries intervening between Europe and India, viz., Asia Minor and Persia. He preferred a railway passing through Sivas and Diarbekir, in Asia Minor (where it would cross the Taurus range of mountains), Mosul, and Kefri, in Turkish Arabia, with a short branch to Bagdad, crossing the Gates of Zagros range on the Persian frontier, and going through the Persian towns of Kermanshah, Hamadan, Teheran, Shahrud, and Mushed, crossing the Afghanistan frontier, and passing through Herat and Candahar and through the Bolan Pass into Shikapore, a station on the Indus Valley Railway. Such a line would open up the most populous and traffically-valuable districts of Persia, towards which country, as well as towards India, Russia is hastening every day.

Colonel Sir Arnold Kembell wished to impress on the meeting the vastness of such an enterprise. He believed it would be of much importance in view of the interests of England and of India, and of great value in opening up the trade of Turkish Arabia.

Mr. Trelawney Saunders said the importance of such a line of railway could hardly be overestimated, and that he thought the route indicated on the map would be eventually that selected for a railway to India.

INEFFICIENT CONTINUOUS BRAKES. — A treacherous friend is more dangerous than an open enemy, and it is far better to be without a continuous brake than to place reliance on one which will fail when it is most wanted. A few gentlemen connected with the Great Northern Railway have for a long time held to the vacuum brake as being perfect, just as Mr. Webb and one or two London and North-Western Railway men claim that the chain brake is the best in the world. Constant practical failures have taught these men nothing. Perhaps because the failures did not entail actual loss of life or limb on a large scale. How

long they are to enjoy this immunity is, however, a doubtful question, and an accident which occurred on Friday afternoon to the Flying Scotchman on the Great Northern line, teaches a lesson which should not be overlooked. As this train, which does not stop between York and Grantham, was passing through Bawtry station, it was noticed by some of the officials on the platform that one wheel of one carriage was off the line, and a telegram was immediately despatched to the next station to stop the train. The carriage, however, caught the bridge over the Scaftworth road, and carried away a portion of the stone coping, and was pulled up at about 200 yards beyond. By this time some six carriages had left the metals and considerable damage was done to the permanent way. The passengers were much shaken, and one lady was seriously injured. Two of the first-class carriages were almost on their sides, while the axle of another was broken. The communication cord was broken before it was noticed that a carriage had left the rails, and the vacuum brake was rendered useless in consequence of its having become disconnected. If the principal features of this accident be contrasted with that of certain others in which the train was fitted with the automatic brake, a vast difference in favor of the latter will be recognized. The vacuum brake will do the work of guards and firemen very fairly, but as a safety brake it is quite inefficient. This truth has long been recognized by everyone, save a few individuals who probably like the vacuum brake because it is the smallest remove from the old hand brake to be found; and if a change must be made, they give the preference to the least change possible. Railway companies require severe lessons to teach them to do what is right. The Great Northern Company narrowly escaped receiving such a lesson on Friday.

ENGINEERING STRUCTURES.

TUBULAR PILES.—The difficulty which attends driving long tubular piles is well known to all those who have attempted to drive them, especially when they are to be driven from the surface of water of considerable depth above the bed of the water course. Messrs. Le Grand and Sutcliffe, of Bunhill-row, so well known through their connection with tube wells, have devised a method of putting down tubular piles of any length. This consists in driving them by means of an elongated cylindrical monkey which works inside the tubular pile. The monkey is raised by means of rods or rope, and allowed to fall upon the flat head of the solid point. The pile thus forms its own guide for the monkey. One advantage of these piles is that they can be driven with facility, of very great length and in deep water. A wrought or cast iron pile, say 75 ft. long, is lowered through the water length by length, screwed together by strong barrel-shaped steel sockets until ground is reached and additional lengths are added as the pile is driven. They can also be filled with concrete well rammed to increase their strength. For some purposes the piles are made in a short length with a flange on the top, the length being sufficient to allow the flange to stand at the ground surface; a tubular or other post with a bottom flange can be fastened thereto permanently or temporarily. It might at first sight appear that a heavy ram falling upon the inside upper surface of the point of a long pile would tend to pull such a pile in two at the screw joints by reason of the inertia of rest of all above the point. It does not seem, however, that this is the case, and short cast iron piles have been driven a number of times, on some occasions into a macadamised road, without apparent injury. The method of driving secured great facility and simplicity, and the number of applications of the method is almost innumerable. For tube wells, piles, telegraph and signal posts, tent poles and plugs, it will be very valuable, and as a tool for making holes for wood telegraph posts, a short tubular pile and hand monkey will greatly simplify operations. The War-office has instructed the Royal Engineer Committee to make a series of experiments with this pile in various soils immediately. The experiments will take the form of actual bridge construction.

THE ST. GOTHARD TUNNEL.—In a paper just communicated to the French Academy, M. Colladon gives some interesting details of the progress of this great enterprise. Besides the excessive hardness of the banks of serpentine and quartz, and the insufficiency of hydraulic forces on the Airolo side (after the lowness of water in winter), there has been a very troublesome infiltration in the south portion, the volume of water having increased since the second year to more than 230 litres per second in the advance gallery. The difficulty of working here under jets which had often the force of those from a fire-engine pump, can be readily imagined. Another difficulty arises from a mass of decomposed felspar mixed with gypsum, found under the plains of Audermatt. This plastic material swells on contact with moist air, and exerts a pressure of terrible force, capable of crushing the strongest woodwork, and even arches of granite. In some of these parts the workers thought themselves happy when they were able to advance (with manual boring) about 1 metre in three or four days; whereas, through granite, with compressed air and mechanical perforation, a regular advance of 4 metres in 24 hours has been achieved through gneiss 6 metres, and more. As regards apparatus, M. Colladon states that the volume of water from the Tremola (Airolo side) having been found insufficient, M. Favre brought water in an aqueduct, 3000 metres long, from the Tessin, to work new turbines of four compressors, on the same system as the others, but of greater value. These turbines are of cast iron. It is noticeable that the old and smaller bronze turbines (formed of one piece), which have made some 155,000,000 revolutions per annum, are in good preservation, after four, or even five years' service, and still work usefully, after slight renewals. On each side of the tunnel there are at present sixteen air compressors in action, serving both for aeration and for boring operations. They send into the tunnel air under a pressure of eight atmospheres, sufficient to drive eighteen to twenty drills, and effect good ven-

tilation of the part already bored, which is at present 6100 metres on the north side, and 5390 metres on the south side. On either side there are, night and day, several hundreds of workmen with lamps, and about 300 kilogrammes of dynamite are consumed. The compressors are found to suffice for good ventilation, rendering unnecessary two large exhaust vessels, placed two years ago at either mouth of the tunnel for drawing off smoke and vitiated air. The transport of materials is effected by horses in the more advanced part of the tunnel, and by compressed air locomotives in the portions near the mouths. To feed these locomotives eight of M. Colladon's compressors are placed at Airolo and Goeschenen. They draw air from the ventilating pipe, and force it, under a pressure of 12 or 14 atmospheres, into a pipe which passes along the part traversed by the locomotives. A great variety of rock drills have been used. Each year brings new improvements.

ORDNANCE AND NAVAL.

THE experimental cruise of the Royal Mail steamer Gallia, already celebrated on account of the part played in her construction by foreign workmen, took place on the 27th ult. On her trial she made the mile in three minutes and forty-five seconds, equal to sixteen knots an hour, or eighteen and one-quarter miles. She is bark-rigged and has nine iron bulkheads, of which seven are watertight, and run up to the spar deck, forming eight watertight compartments. There are two dining saloons for cabin passengers, one of spacious size, on the spar deck. It is lighted by a handsome cupola, and beautifully ornamented with Japanese panelling. The coal bunkers are well ventilated and constructed to contain 1,000 tons of fuel. The water tanks are made of galvanized iron, and are capable of holding 14,000 gallons. The fresh water condensers are capable of providing 4,000 gallons daily. Steam steering gear is fitted amidships, but to meet casees of emergency there is also a manual steering gear with wheel-house aft. There are iron lighthouses, entered from the deck below, allowing lights to be easily accessible even in the wildest weather. Suspended from davits are ten large lifeboats, with patent lowering apparatus. The Gallia is the largest and most complete of all the Cunard ships. She starts on her first voyage to New York to-morrow.

KRUPP'S GREAT STEEL GUN.—Herr Krupp is about to try at Meppen, Germany, the latest and largest steel gun turned out by his great works, and indeed the largest specimen of steel ordnance yet made. It weighs 72 tons, with a calibre of 15¾ in. The length of the gun is 32 ft. 8 in., and that of the bore 28 ft. 6 in. The English 80-ton gun has a calibre of 16 in., a total length of 27 ft., and a bore 24 ft. long. The charge for Krupp's monster gun is to be 385 lb. of prismatic powder, the projectile being a chilled iron shell, weighing 1,660 lb., and having a bursting charge of 22 lb. of powder. The force of the shot on leaving the gun is estimated at 31,000 foot-tons, and it is calculated that the gun can throw its projectile a distance of 15 miles. The forthcoming trials will take place on a range 11 miles long, and the targets will have to be placed at such a distance that the gun will have to be directed by other means than the visibility of the object to be struck. The material of which the Krupp gun is composed is steel throughout. The core of the gun consists of a tube running its entire length, as in the Woolwich gun, but open at the rear, the loading being at the breach instead of the muzzle. The tube of this large weapon being of such great length, it has been made in two portions, the joint being secured in a peculiar manner. Over the tube are four "jackets" or cylinders of various lengths, supplemented by a ring over the breech portion. The cylinders are much less massive than in the Fraser gun, and approximate more to the pattern of the Armstrong ordnance. The gun is chambered, that is to say, the powder chamber has a greater diameter than the bore. The form given to the powder prisms, and the adjustment of the cartridge in the bore, allow altogether an amount of space which gives 40 per cent. of air to the powder actually composing the charge. The rifling is polygonal with a uniform twist, and the shot is rotated by means of a copper ring around its circumference near the base, the Vavasseur system in short. The closing of the breech is effected by the well-known Broadwell system adopted for all his ordnance by Mr. Krupp.

BOOK NOTICES

MODEL YACHT BUILDING AND SAILING. London: Charles Wilson. For sale by D. Van Nostrand. Price $2.00.

This subject possesses rare attractions for a large class of people who delight to pursue a diversion with a systematic plan and scientific precision.

This is a complete instruction book for the amateur model yacht builder and includes directions for all details. It is also supplemented with an illustrated vocabulary of nautical technology.

The author has given his instruction in a singularly felicitous straightforward way which will surely prove acceptable to all interested in the subject.

ELECTRIC LIGHTING AND ITS PRACTICAL APPLICATION By J. N. SHOOLBRED, M. I. C. E. London: Hardwicke & Boyne. For sale by D. Van Nostrand. Price 2.50.

This is a very compact summary of descriptions of present modes of electric lighting, and affords a fair estimate of the relative value of machines and lights which are now before the public.

The chapters treat severally of topics as follows:

I. Historical and Descriptive. II. Electric Light Machines. III. Lamps and Regulators. IV. Carbons and Conducting Wires. V. Motive Power. VI. Luminous Intensity. VII. Applications and Economic Results. VIII. Prospects of Electric Lighting.

The illustrations are sufficiently numerous and of fair quality,

THE STUDENT'S TEXT-BOOK OF ELECTRICITY. By HENRY M. NOAD, F. R. S. A new edition with additional chapters. By W. H. PREECE, M. I. C. E. London: Crosby, Lockwood & Co. For sale by D. Van Nostrand. Price $5.00.

This new edition of an excellent work is quite timely. Nothing better than the earlier edition has ever appeared to meet the wants of those who apply electricity. Nothing else in English is so full of practical electrical science.

This new edition is enriched by the additions of Mr. Preece, who has given a summary of dynamo-electric inventions and electric illumination, down to the beginning of the present year.

The remarkable abundance of illustrations which characterized the former edition is exhibited quite as fully in the new chapters.

COAL MINES INSPECTION—ITS HISTORY AND RESULTS. By R. NELSON BOYD, F. R. G. S. London: W. H. Allen & Co. For sale by D. Van Nostrand.

This is a history of the legislation in Great Britain relating to the inspection of coal mines, giving a full account of the improvements in methods and machinery adopted from time to time, whereby the danger to life in the mines has been steadily lessened.

It appears from the statistics given quite fully in the appendix, that whereas, in 1851, the mining of one million tons of coal cost 19.35 human lives, in 1876 the same amount of coal cost but 6.95 lives, and that the decrease was steadily maintained during this period.

To the English collier, or the mine owner, the book is full of interest.

LES METAUX A L'EXPOSITION UNIVERSELLE DE 1878. Par H. SEBASTEUR Paris: Dunod. For sale by D. Van Nostrand. Price $5 00.

This is a large quarto volume of 300 pages of text and 16 pages of plates.

The metals treated of are the various irons and steels together with such alloys of copper as have been proved to possess an exceptional strength.

The resisting powers of the metals is treated at considerable length as is also the method of determining such resistances by the various testing machines. Most of the illustrations are of these machines and include the prominent and familiar American forms.

The work is a valuable compendium of information on the subject of metals and in engineering constructions.

A PRACTICAL TREATISE ON THE COMBUSTION OF COAL. By WM. M. BARR. Indianapolis: Yohn Brothers. For sale by D. Van Nostrand. Price $2.50.

This work aims to present within a moderate compass the theory of the combustion of coal, with a view of adapting it to the needs of that large body of men whose interests are identified with the use of coal, and to whom a correct knowledge of the subject is especially desirable, but who, on account of the abstruse style of the present treatises, cannot readily obtain the required information.

All that the author claims is the adaptation of the science of the higher treatises to readers who find difficulty in studying the higher standard works.

Considerable space is devoted to descriptions of special devices for burning fuel of different kinds. The illustrations are of fair quality, and the general typography is excellent.

THE TRANSMISSION OF POWER BY COMPRESSED AIR. By ROBERT ZAHNER, M.E. New York: D. Van Nostrand. (Van Nostrand's Science Series, No. 40). Price 50 cts.

The opinion is now very general that the maximum of economy in the use of rock-drilling machinery is to be looked for less in the further improvement of the drills themselves than in the perfection of the air-compressing machinery, and this perfection can only be arrived at by careful attention to the various scientific considerations connected with the subject. The whole question is ably discussed in a recent volume of Van Nostrand's Science Series, by Mr. Robert Zahner, and the information he supplies is so valuable that it should be generally studied by all who have occasion to use compressed air. The subject of compressed air and compressed-air machinery offers, as Mr. Zahner states, a wide field for useful investigation. Compressed air has become a most efficient and powerful agent in the hands of the modern engineer. Its applications are rapidly growing both in extent and importance. There can be no doubt that the great waste of energy that to-day accompanies the use of compressed air is due not only to sickly design and faulty construction of machines, but very largely also to the general ignorance of the principles of thermodynamics. In his historical notice Mr. Zahner points out that the application of compressed air to industrial purposes dates from the close of the last century. Long before this, indeed, we find isolated attempts made to apply it in a variety of ways, but its final success must be ascribed to the present age—the age of mechanic arts—an age inaugurated in so splendid a manner by the genius of Watt, and which has been so wonderfully productive in good to mankind. Cubitt and Brunel, between 1851 and 1854, first applied compressed air in its statical application to the sinking of bridge caissons. Prof. David Colladon, of Geneva, in 1852, first conceived and suggested the idea of employing it in the proposed tunneling of the Alps, and finally Sommeiller first practically realized and applied Colladon's idea in the boring of the Mont Cenis Tunnel.

For transmitting power to great distances shafts, belts, friction wheels, and gearing are clearly out of the question The practical incompressibility and want of elasticity of water renders the hydraulic method unfit for transmitting regularly a constant amount of power; it can be used to advantage only where motive power acting continuously is to be accumulated and applied at intervals, as for raising weights, operating punches, compressing, forging, and other work of an intermittent character requiring a great force acting through a small distance. Compressed air is the only general mode

of transmitting power; the only one that is always and in every case possible, no matter how great the distance nor how the power is to be distributed and applied. No doubt as a means of utilizing distant yet hitherto unavailable sources of power the importance of this medium can hardly be overestimated. But compressed air is also a storer of power, for we can accumulate any desired pressure in a reservoir situated at any distance from the source, and draw upon this store of energy at any time, which is not possible either in the case of steam, water, or wire-rope. But compressed air is especially adapted to underground work; steam is here entirely excluded, for the confined character of the situation and the difficulty of providing an adequate ventilation renders its use impossible. Compressed air, besides being free from the objectionable features of steam, possesses properties that render its employment conducive to coolness and purity in the atmosphere into which it is exhausted. The boring of such tunnels as the Mont Cenis and St. Gothard would have been impossible without it. Its easy conveyance to any point of the underground workings, its ready application at any point, the improvement it produces in the ventilating currents, the complete absence of heat in the conducting pipes, the ease with which it is distributed when it is necessary to employ many machines whose positions are daily changing, such as hauling engines, coal-cutting machines, and portable rock drills; these and many other advantages when contrasted with steam under like conditions give compressed air a value which the engineer will fully appreciate.

There is every reason to believe that compressed air is to receive a still more extensive application. The diminished cost of motive power when generated on a large scale compared with that of a number of separate steam engines and boilers distributed over manufacturing districts, and the expense and danger of maintaining an independent steam power for each separate establishment where power is used, are strong reasons for generating and distributing compressed air through mains and pipes laid below the surface of the streets, in the same way as gas and water are now supplied. Especially in large cities would the value of such a system be invaluable; no more disastrous boiler explosions in shops filled with hundreds of working men and women; the danger of fire greatly reduced; a corresponding reduction in insurance rates; an important saving of space; cleanliness, convenience, and economy. As affording a means of dispensing with animal power on our tramways compressed air has been proposed as the motor. It has already met with some success in this direction, and to-day there are eminent French, English, and American engineers at work upon this interesting problem. Mr. Zahner treats first of the conditions modifying efficiency in the use of compressed air—loss of energy, methods of cooling, conditions most favorable to economy in the use of compressed air, efficiency attained in practice, and losses of transmission. In treating of the physical properties and laws of air Mr. Zahner deals with the subject in a manner which will make it intelligible to every practical man, and the same may be said of the subsequent chapters—thermodynamic principles and formulæ, thermodynamic equations applied to permanent gases, thermodynamic laws applied to the action of compressed air, the efficiency theoretically attainable, the effects of moisture, of the injection of water, and of the conduction of heat; American and European air compressors; and examples from practice. There is, probably, no other book containing the same amount of useful information in a similar space, a circumstance which will suffice to commend it to the attention of every engineer who uses or could use compressed air as a motor.—*London Mining Journal*

MISCELLANEOUS.

MEAN DEPTH OF THE SEA.—A large amount of material for arriving at some approximately correct notion of the mean depth of the sea, has accumulated in recent years. In a note to the Göttingen Academy, Dr. Krümmel has lately attempted this, in view of the vague and variable statements on the subject in text-books. Soundings were wanting for the Antartic and a part of the North Polar Sea, *i. e.*, about 475,000 square miles, or 7 per cent. of the entire sea-surface, so that he gives his estimate only as a closer approximation. He estimates, then, the mean depth of the sea as 1,877 fathoms, or 3,432 meters, or 0.4624 geographical miles. It was natural to compare the mean height of dry land above the sea level. Humboldt's estimate of 308 meters is regarded as quite out of date. Leipoldt has since estimated the mean height of Europe as 300 meters. Accepting this number for Europe, 500 for Asia and Africa, 330 for America, and 250 for Australia, Dr. Krümmel obtains the mean of 420 meters, or 0,0566 miles. The surface-ratio of land to water being considered 1:2.75, the volume of all dry land above the sea-level is inferred to be 140,086 cubic miles, and the volume of the sea 3,138,000 cubic miles. Thus the ratio of the volumes of land and water is 1:22.4. That is, the continents, so far as they are above the sea-level, might be contained 22.4 times over in the sea-basin. Reckoning, however, the mass of solid land from the level of the sea-bottom, the former would be contained only 2.443 times in the sea-space. Dr. Krümmel also compares the masses (taking recent data); he finds that of the sea 3,229,700 cubic miles, and that of the solid land 3,211,310 (a small difference). If the specific gravity of the land were raised merely from 2.5 to 2.51432, we should thus have perfect equilibrium. Such equilibrium is probably the fact.—*Nature.*

ONE of the latest attempts to correct the irregular flight of war rockets consists of fixing an additional head to the missile with holes in the neck for the escape of the powder gas rearwards, similar to the escapement at the base. It was thought that this would cause the rocket to preserve a more correct balance as well as increase its range, but it has found in practice to have the contrary effect, and the suggested improvement is a failure.